Three Types of Interaction in Multi-Species Fisheries and When They Need to be Considered

Benjamin Blanz
University of Hamburg
benjamin.blanz@uni-hamburg.de

January 2018

Abstract

Management of multi-species fisheries is made complicated by interaction between the different species involved. Interaction between species may take place within the ecosystem, through simultaneous inseparable harvesting or through consumer demand. While each of these types of interactions has been shown to be significant individually, analyses including all three are lacking. In this paper an analytical model of multi-species fisheries is used to determine optimal harvesting rates incorporating all three types of interactions. This is done in order to determine the consequences of omitting individual types of interaction and to investigate possible inter-dependencies. Furthermore their importance in the design of total allowable catch and quantity tax based management is investigated. While ecosystem interactions between species are almost trivially important in setting optimal harvesting quotas, the significance of the other types of interaction is less obvious. Depending on the goals of the manager, their specific properties and the management method they may be omitted.
Keywords: Multi-Species Fisheries, Dynamic Modelling, Market Incentives, Compliance,
Ecological-economic systems

1 Introduction

Through fishery human actors have a direct impact on aquatic ecosystems. This impact may be compounded by interactions between multiple species within these ecosystems. Furthermore, fisheries are often indiscriminate, impacting multiple components of the ecosystem simultaneously, but even when selective harvests are possible human actors may prefer to consume harvests of the individual species simultaneously in certain proportions. Management of fisheries then is the attempt to control this human impact on fish stocks and the wider ecosystem of which they are a part and which sustains them. The goals motivating implementation of management measures may simply be to achieve maximal (sustainable) harvest rates, to sustain stock levels of a certain species, to ensure that biodiversity is maintained or to ensure that some other measure of ecosystem health is met. In short, the job of the fisheries manager is to ensure that fishery effort is based on society's preferences and not only those of fishers, by implementing appropriate laws and regulations.

Depending on the goal of the manager, the extent of knowledge required with respect to the managed ecosystem, but also of fisheries and societal preferences, is variable. To aid in structuring the known properties of the managed ecosystem and possibly its interaction with harvesters and consumers various models are used (Hollowed et al., 2000; Link, 2002b). These range from very simple representations of single species with exogenously set harvests (Pearl and Reed, 1977) to highly complex descriptions of fisheries and related ecosystems including as many details as possible (Pelletier et al., 2009). In any case, models are needed to organize and represent available data and systemic knowledge of the relevant systems. They are further needed in order to derive management measures from the available data.

Simple models have the benefit that they can be analytically solved in order to find general results that can be used across a wide variety of ecosystems, such as the maximum sustainable yield (MSY) of a given harvested species. However, excessive simplification may cause these general results to be of little use in management, when factors not included in the model may cause an ecosystem managed on such a simplistic model to fail. Harvest rates deemed sustainable may be fatal when cross impacts and ecosystem feedbacks are considered. Conversely, what is deemed the maximum sustainable harvest in a single species setting may not be the actual maximal sustainable harvest rate if positive or negative feedbacks in species growth where omitted (Ströbele and Wacker, 1991; Pikitch et al., 2004). Consequently, these types of models have long been considered too simplistic to be used as the foundation of management (Larkin, 1977).

Highly complex models, meanwhile, have significantly larger data and computational requirements in order to yield useful results (Link, 2002b). The necessary data to correctly calibrate and run such a model ranges from moderately difficult to practically impossible to obtain. Whereas prices of sold fish can be observed in the market with limited effort, estimating the state of the ecosystem or harvesting properties requires costly research vessels and operating expenses. Estimating consumer preferences and determining reactions in demand due to changing prices is an area of much research within economics, which has proven to be anything but simple. Furthermore, increasing complexity is the bane of tractability, implying that it is difficult to analytically derive general results using such models.

A basic rule when developing a model is to include as much complexity as is necessary, while keeping the model as simple as possible. The aim of modelling after all is to create parsimonious models that still have a high explanatory power. To aid in answering the question how much complexity is necessary, I attempt to give some indication as to what types of interactions between species need to be included in models of multi-species fisheries and which may be omitted in the name of simplicity. The three types of interaction between harvested species I consider are interactions between species within the ecosystem, technological interactions in harvesting between species and interactions between demanded quantities of different species by the final consumers of harvested fish. Each of these types of interaction has a large literature describing their individual importance for management. However, attempts to investigate possible inter-dependencies between them, causing positive or negative feedbacks between harvesting rates, appear to be quite rare. These types of interactions between interactions between species are of special interest, as they may cause unexpected behaviour in a system that was thought to be successfully managed. This is especially true for the human components, fishers and consumers, of the coupled ecological economic fisheries system (e.g. Fulton et al. (2011)).

Each of the avenues of interaction between the harvesting of different species, ecosystem interaction, technological interaction in harvesting and demand side interaction due to consumer behaviour change the way optimal management is conducted.

The first type of interaction, between species within the ecosystem, determines the capacity of the ecosystem to recuperate from harvesting. If harvests exceed this capacity they will not be sustainable. Crucially, if ecosystem interaction is present, the capacity of the ecosystem to recoup lost stocks depends on the composition of species remaining. Link (2002a) derives a lengthy set of questions, based on which a modeller may determine the importance of ecosystem interactions for the system to be modelled. Ecosystem interactions cause changes in one stock to have a feedback effect on another. A classical example of such feedbacks are predator-prey relationships between individual species. The impact of such relationships for management has been investigated by e.g. Yodzis (1994). But more complex relationships between more than two species may also need to be considered (May et al., 1979). In the same context Plagányi et al. (2014) describe that it would be unwise to manage multiple species in the same ecosystem as if they were independent. Various case studies have been performed illustrating the importance of considering ecosystem interactions for the fisheries modelled (e.g. Gulland and Garcia (1984)).

The second type of interaction, technological interactions in harvesting, also called bycatch, cause harvests of one species to be associated with simultaneous unavoidable harvests of another species. Impacts on parts of the ecosystem which are not harvested are also possible. The simultaneous unintended catch of a different species (Skonhoft et al., 2012; Nieminen et al., 2012) or age group (Davies et al., 2009) than that which intended is termed bycatch. Bycatch is caused by fishers not being able to perfectly select which species are harvested or by selectivity being costly (Abbott and Wilen, 2009; Singh and Weninger, 2009). However, fishers will not necessarily try to avoid bycatch. In multi-species fisheries the species caught as bycatch will often also have market value. In this case it will be landed and marketed and can be readily be included in management. Bycatch not brought to market however implies that fishers discard part of their catch at sea. This increased impact on ecosystems, over that caused by marketed quantities from discarding, is difficult to estimate due to reliance on self reported data by fishers. Davies et al. (2009) estimate that at least 40 % of total fishing mortality is not due to marketed harvests but discarding. The incentives of fishers to discard are determined by the relative contribution of caught species to individual profits. These critically depend on market prices and quantities demanded by consumers or set by regulation.

The third type of interaction, demand side interactions between different species are caused by the preferences of final consumers for fish products of individual species and their relative amounts. These preferences are reflected in the household demand functions which constitute the demand side of the market for fish products. It has been shown that consumer preferences may have a significant impact on the state of the ecosystem by driving harvests through their impact on prices (Baumgärtner et al., 2011). In light of empirical evidence (Barten and Bettendorf, 1989; Bose and McIlgorm, 1996; Asche et al., 1997; Chiang et al., 2001) that different species of fish are viewed as imperfect substitutes by households Quaas and Requate (2013) show that even when species are independent in their ecological and harvesting properties, substitution between species by consumers may lead to sequential overfishing of all available species.

Determining appropriate management measures is further complicated by the fact that these different avenues of interaction between species are not independent of each other. Changing ecosystem stocks impact harvesting rates which impact prices, prices in turn incentivise fishers to adjust harvesting rates which in turn impact ecosystem stocks, either amplifying or dampening this cycle. Each of these avenues of interaction causes feedbacks between the stocks of individual species. These feedbacks may strengthen the effect of interaction between species or compensate each other, thereby weakening the interaction. It is even possible that different avenues of interaction cancel each other out with regards to the actual impact on the ecosystem. Such a case is investigated by Blanz (2018). Therein a condition is derived under which changing amounts of bycatch/technological interaction have no effect on harvest rates. This is possible as harvest rates depend not the only on the harvesting efficiencies of the different gear types but also on the size of the respective fleets using the respective gear types. Thereby, it is possible that scenarios with different gear effectivities obtain identical results with regard to harvest rates. The different harvesting efficiencies are perfectly offset by changes in fleet composition. To explain these changes in fleet composition, market demand needs to be included in the model. Market demand adapts in response to the changes in supply, caused by changed catch composition. However, the conditions necessary for this effect to perfectly cancel out impacts from technological interaction between ecosystem stocks are very narrow, implying that in the general case technological interaction between species in conjunction with market forces appears to be especially relevant to determining the effectiveness of management measures. Squires et al. (1998) investigate compliance with individual transferable quota (ITQ) based management schemes and find that fishers have an increased incentive to discard part of their catch, when the relative proportions of harvested species do not match those prescribed by the ITQ. A similar issue is described by Beddington et al. (2007). Regarding incentives of individual fishers to discard part of their catch when regulated by quotas, Abbott and Wilen (2009) model the choice problem of fishers in a game theoretic context. They find that the decision maker is severely restricted in choosing harvesting quotas, if large amounts of discards are to be avoided. As an alternative to management using quotas I describe the implementation of management through taxes on quantities, which can either be applied to prices paid by consumers or as a landing fee for harvesters.

In the context of all these interactions and their inter-dependencies that potentially need to be considered when designing effective management measures, the focus of this paper are the consequences of the inter-dependency of technological and demand side interaction for the design of management measures. To this end the socially optimal harvest rates are analytically determined, while taking into account ecosystem, technological and demand side interactions between species. The implementation of these harvesting rates through different management measures is then analysed to determine their effectiveness, given the three types of interactions and their inter-dependencies.

The remainder of the paper is structured as follows: In the following section (Section 2) the model equations characterising the ecosystem, harvesters and household preferences, each including the possibility for interaction between species, are presented. Furthermore the market equilibria, resulting from the interaction of harvesters profit maximizing behaviour and consumer demand or socially optimal demand, are derived. The implementation of different management measures to achieve the derived socially optimal harvesting rates and investigation of their expected effectiveness given the behaviour of the human actors are performed in Section 3. The results of these efforts, with respect to the three types of interactions between species described in this paper, as well as limitations of the analysis are discussed in Section 4. Concluding remarks are given in Section 5.

2 The Model

In order to demonstrate the combined effect of the three types of interaction between different species the model needs to incorporate each type and allow interaction between the components containing the respective types of interaction. Tractability of the model requires limiting the analysis to two species and harvesting gear types. While this prohibits analysing cases where entire foodwebs are relevant to setting optimal harvest rates, other types of ecosystem interaction can be considered, regardless. It is expected that results obtained in this reduced model will apply analogously to cases with more species and harvesting tools.

The components of the model are shown in Figure 1 and are presented in greater detail in the subsections below. The

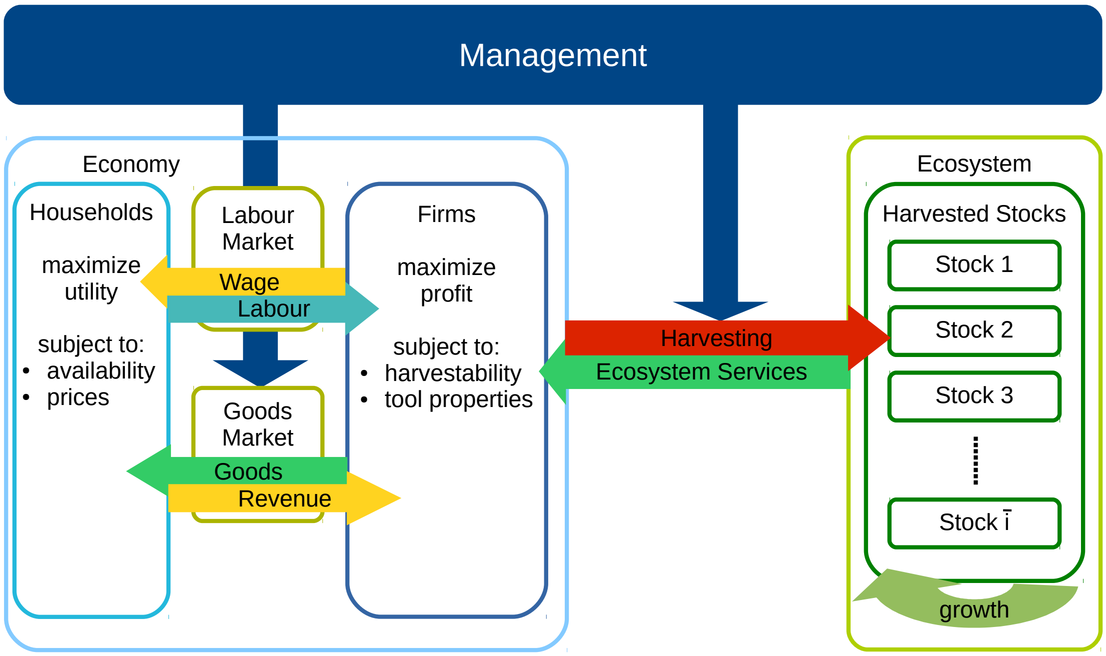

Figure 1: Components of the model and their interactions. Within each component species stock levels or their corresponding harvest rates may interact. Harvesting is performed using multiple different gear types, each with different harvesting efficiencies for each of the species included in the model.

model consists of an ecosystem component, describing its current state and dynamics, harvesting firms, maximizing profits, and consumers, maximizing contemporaneous utility. Between harvesting firms a market for goods allows for the sale of harvested ecosystem stocks to consumers. The prices on this market and corresponding harvested quantities are determined endogenously. A second market for labour allows firms to employ labour provided by households in harvesting or manufacturing of a numeraire commodity, thereby providing income to households, needed in order to pay for the fish and other products consumed. Management may either influence harvested quantities directly or through taxes on the goods market. As it is an analytical model the components are kept as simple as possible, in order to have tractable solutions, while still reproducing all of the avenues of interactions between species discussed in the introduction.

In order to include ecosystem interaction, the stock growth equations include interspecies competition. While this does not represent more involved predator prey interactions, it is sufficient to ensure that the partial derivatives of stock growth of one species to the others is not zero. This is required in order for the optimal harvesting rate to also depend on the growth of all stocks. With this present, the exact form that the ecosystem equations take is not relevant for the general results as to the importance of ecosystem interaction in management.

Technological interaction in harvesting is implemented through catch efficiency vectors for each of the available harvesting gear types. Thereby each gear has a parametrised catch efficiency for all available species. This allows investigation of cases with or without bycatch in a general manner. This also allows determining conditions on when technological interactions can be ignored as they are perfectly compensated for by other effects. Fisher behaviour is modelled as that of profit maximizing firm endogenously determining optimal harvesting rates given demand and market pressures.

Interaction between species through the demand side of the fish market is created by household preferences that allow for a parametrised limited degree of substitution between different species in consumption. From these the household demand function is derived, which relates demanded quantities of a specific species to the prices of all available species. If the price of another species falls, the household may substitute consumption of the cheaper species for that of the more expensive one.

The model was extended from Quaas and Requate (2013) to include technological interaction in harvesting and incorporate ecosystem interaction as in Baumgärtner et al. (2011) by Blanz (2018) and solved for an open-access setting without any management. In this paper that analysis is further extended by the inclusion of a social welfare function aggregating household utility over time and derivation of socially optimal harvesting rates. The equations and derivations in this section are reproduced from Blanz (2018) in order to provide a foundation for the following sections.

In the remainder of this paper the sets of species and the set of harvesting gear types included in the model are given by I and K respectively. Species are indexed by i and harvesting gear by k. As an example, the stock of species i is given by x_i and harvesting efficiency for species i using gear type k is given by ν_{ik}.

$$I = \{1,2\} \quad K = \{1,2\} \tag{1}$$

2.1 Ecosystem Dynamics

Each of the species in the system is represented by a stock variable tracking the current biomass relative to the carrying capacity of the ecosystem for that species. The state of the ecosystem at time t is given by the vector $\vec{x}_t$ of length $\bar{i}$ with entries x_i for each species. Stock change (2) is determined by the difference of intrinsic growth of the species, depending on the state of all species, and total harvests.

$$\dot{x}_{it} = g_{it}(\vec{x}_t) - H_{it}(\vec{x}_t) \tag{2}$$

The mode includes a logistical growth function for the intrinsic stock change component g_{it}. However, the analytic results derived below do not depend on that exact shape of the intrinsic growth function, only whether the cross derivatives between species are zero. For the equation below this is the case whenever the matrix γ containing the interspecies competition vectors $\vec{\gamma}$ is not equal to the identity matrix and contains non-zero elements off the diagonal.

$$g_{it}(\vec{x}) = r_i(x_i - \underline{x}_i)\left(1 - \frac{\vec{\gamma}_i \vec{x}_t}{\kappa_i}\right) \tag{3}$$

$$\frac{\partial g_i(\vec{x})}{\partial x_{i'}} \neq 0 \quad i \neq i' \quad i, i' \in I \tag{4}$$

According to the logistic growth function (3) a stock will grow as long as it is above the species specific minimum viable biomass threshold $\underline{x}_i$ until it reaches the carrying capacity of the ecosystem for this species κ_i which may be shared with other species depending on the per species competition described by γ_{ij}, the elements of the species specific interspecies competition vector $\vec{\gamma}_i$. In the case with no interspecies competition $\gamma_{ij} = 0 \; \forall \; i \neq j \; \in I$.

The stock change and optimal management equations are the only time dependent components of the model. All other equations of the model depend only on the current state of the ecosystem. In the remainder of the paper any state variable without a time index t is defined to be contemporary.

2.2 Harvesting

Total harvests per species H_i are determined by the size of the fleets n_k operating harvesting gear k in the set of harvesting gear types available K and the harvest of that species per vessel using each gear type $h_{ik}(e_k, x_i)$. It is assumed that for each species in the model there is one gear type targeting it, with the possibly of simultaneously catching other species as bycatch. Consequently, there are $\bar{k} = \bar{i} = 2$ gear types in the model. This assumption is based on the logic that if there were more gear types than species available, only the most efficient would be used, depending on which species is targeted. Fishers using the more efficient gear could undercut prices of those using less efficient gear. Other fishers would either leave the market or change their gear type. In the model such changes in gear type are reflected as changes in the fleet composition.

$$H_i = \sum_{k=1}^{\bar{k}} n_k h_{ik}(e_k, x_i) \tag{5}$$

Harvests of species i per vessel using gear k depend on the stock dependent availability $\chi_i(x_i)$ of species i in combination with the gear specific harvesting efficiency ν_{ik} and the harvesting effort e_k. The additional catch for each additional unit of effort depends on the returns to effort ε. Stock dependence of harvesting is described by $\chi_i(x_i) = x_i^{\chi_i}$ where a specific form is needed. In the remaining sections $\chi_i(x_i)$ is abbreviated as χ_i, while χ indicates a square matrix with the χ_i along its diagonal.

$$h_{ik}(e_k, x_i) = \chi_i(x_i)\nu_{ik} e_k^{\varepsilon} \tag{6}$$

The harvesting effort e_k is determined endogenously for each gear type. Fishers are modelled as profit maximizing firms each operating a single fishing gear k with an endogenously determined amount of effort e_k. Each fishing gear is intended to catch one of the modelled species as its target but may also catch any other species, depending on the gear specific harvesting efficiencies ν_{ik} for each of the species i. All fishers operating a specific gear k are assumed to be identical. The fishers' profit function is given by the difference between revenues from selling all fish caught using a selected gear and total costs. Costs consist of fixed gear specific costs χ_k and variable costs depending on harvesting effort ω.

$$\max_{e_k} \sum_{i=1}^{\bar{i}} h_{ik}(e_k, x_i) p_i - \omega e_k - \phi_k \tag{7}$$

Maximizing the gear specific profit function yields the profit maximal effort dependent on the vector of current prices $\vec{P}$ and harvesting properties. In the case without bycatch, optimal harvesting effort per tool would only depend on the species it is intended to catch, as the elements of $\vec{\nu}_{k \neq i}$ would be equal to zero.

$$e_k^{**}(\vec{P}) = \left(\frac{\varepsilon \vec{P}^{\intercal} \chi \vec{\nu}_k}{\omega} \right)^{\frac{1}{1-\varepsilon}} \tag{8}$$

In the case that perfect competition exists on the fisheries market, profits would be driven to zero. Any fisher achieving positive profits could decrease prices in order to take a larger portion of the market. Zero profits in combination with profit maximal effort yield the zero-profit optimal effort level e_k^* depending only on cost parameters and returns to effort.

$$e_k^* = \frac{\phi_k}{\omega} \frac{\varepsilon}{1-\varepsilon} \tag{9}$$

Given the behaviour of each of the individual vessels using a certain harvesting gear, total harvests are determined by the size of the respective fleets using each gear type, as stated in the beginning of this section. Fleet sizes are determined by the balance of supply and demand on the market for fish. This is formalised in the goods market clearing condition.

$$q_i(\vec{p}) = H_i = \sum_{k=1}^{\bar{k}} n_k h_{ik}(e_k, x_i) \tag{10}$$

Total supply of fish of each species must equal demand for that species $q_i(\vec{p})$. If harvests are not completely independent, i.e. bycatch is present in harvesting, the possible consumption levels q_i are limited by harvesting gear selectivity and non-negativity of fleet sizes.

$$n_k \geq 0 \, \forall k \in K \tag{11}$$

Hence, the supply side of the market for fish is limited to positive linear combinations of the gear specific harvesting vectors $\vec{h}_k(e_k, \vec{x})$ containing the $h_{ik}(e_k, x_i)$ as elements. For strictly positive combinations ($n_k > 0 \, \forall k \in K$) the supply side can provide any composition of species. For other cases, where not all gear types are in use, the species composition of supply is fixed to that of the catch using the remaining gear $\vec{h}_k(e_k, \vec{x})$. In these edge cases, consumer choice is limited to the number of bundles containing the fixed ratio of harvested species. This restriction in harvestable catch composition is illustrated in Figure 2 for the case with two gear types $K = 1, 2$ targeting two species. Total harvest rates for each species H_1 and H_2 are depicted along the axes. The degree of bycatch in harvesting defines the angle of each of the gear harvesting vectors. In the case without bycatch they would lie along the axes, in combination spanning the entire harvest space. With bycatch however, given that negative fleet sizes are not possible, combinations of the harvesting vectors only span a subspace of species compositions in harvest. This will become relevant in the household consumption and optimal harvesting decisions below. Areas outside of the spanned space can only be reached if parts of the catch are discarded. If these harvest combinations are sufficiently more profitable than selling the harvestable catch compositions, discarding becomes economically attractive.

2.3 Households

The demand function for fish and optimal harvesting rates are derived from the household preferences represented in the household utility function of a single representative household. Utility is gained from the sub utility for consumption of fish Q and consumption of a composite manufactured good y representing all other consumption. The

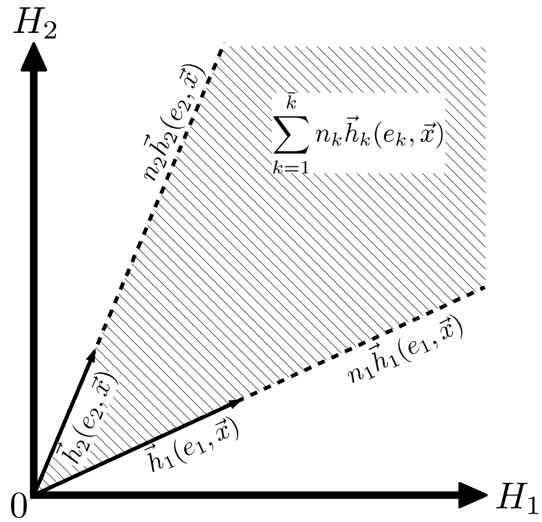

Figure 2: Possible catch compositions of harvested species for the case with two harvesting gear types and two species. Gear types 1 and 2 have species 1 and 2 as their targets, catching the respective other species as bycatch. Given positive stocks of both species, harvest of only one species is impossible. The degree of bycatch determines the angle of the harvesting vectors. The size of the fleets n_1 and n_2 using each of the tools determine total harvests within the shaded area. Combinations of harvests outside the shaded area are only possible by discarding part of the catch.

relative importance of fish consumption compared to other consumption is measured by $\alpha \geq 0$, the elasticity of fish consumption is by $\eta > 0$.

$$U(Q,y) = \begin{cases} y + \alpha \dfrac{\eta}{\eta - 1} Q^{\frac{\eta-1}{\eta}} & \text{for } \eta \neq 1 \\ y + \alpha \ln Q & \text{for } \eta = 1 \end{cases} \tag{12}$$

Preferences for the consumption of individual fish species are modelled as a Dixit-Stiglitz utility function (Dixit and Stiglitz, 1977). This is characterised by a constant elasticity of substitution between different species $\sigma > 0$. Higher values of σ indicate that individual species are better substitutes for each other, the households care less about the composition of consumed fish products and more about the quantity.

$$Q = Q(\vec{q}) = \left(\sum_{i=1}^{\bar{i}} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{13}$$

Household consumption decisions are limited by the budget restriction. The representative household receives income ω from providing labour to either the fishery sector or the manufacturing sector. The amount of labour provided by the household is normalised to unity. The wage rate ω is assumed to be equal across both sectors. Income is spent in order to purchase fish products q_i at market prices p_i or manufactured goods which take the role of numeraire commodity, with unity prices.

$$\omega = y + \sum_{i=1}^{\bar{i}} p_i q_i \tag{14}$$

From these equations the demand function for each fish species depending on prices of all species in the market can be derived. In so doing the inverse demand function can also be derived, relating the willingness to pay for a quantity of a specific species, given the quantities consumed of the other species. These derivations are reproduced in Appendix

B.1 and B.2 respectively.

$$q_i = \alpha^{\eta} p_i^{-\sigma} \left(\sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma} \right)^{\frac{\sigma-\eta}{1-\sigma}} \tag{15}$$

$$p_i = \alpha q_i^{-\frac{1}{\sigma}} Q^{\frac{\eta-1}{\eta} - \frac{\sigma-1}{\sigma}} \tag{16}$$

However, whenever harvests of the individual species are not independent due to bycatch, this may further restrict consumer consumption choices. As is described in the previous section, in this case fisheries are not able to supply all possible combinations of fish quantities. In the edge case where consumers prefer the combination of species harvested using one of the gear types to all other quantity combinations, consumers do not choose individual amounts but instead choose the number of bundles with that combination of species to consume. The reformulation of the utility function to consider bundles of fish quantities and the derivation of the corresponding demand function is shown in Appendix B.4. Demand is then no longer expressed as a set of functions relating individual consumed quantities of species to market prices, but a single function relating the number of bundles consumed to costs required a single bundle and the species composition of that bundle. Given the assumed goods market clearing, the number of bundles demanded (17) will be equal to the fleet size using the relevant gear and the species composition of the bundle is given by the catch composition of the same gear.

$$n_k = c_k^{-\eta} \alpha^{\eta} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \tag{17}$$

The condition to switch from the free to the restricted demand function depends on the degree of substitution between species as well as the stock specific and gear specific harvesting efficiencies. Its derivation from the household optimisation problem is shown in Appendix B.5.

$$0 > \sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}} \left(\frac{\nu_{i1}}{\nu_{i2}} - 1 \right) \tag{18}$$

2.4 Manufacturing Sector

Finally, the model is closed by determining the amount of the numeraire commodity available to consumers. To simplify the analysis it is assumed that the numeraire commodity, the manufactured good y is produced with labour as the sole input at a constant productivity equal to the wage rate. A perfect labour market is assumed ensuring that the wage rate ω is equal across all sectors and equal to marginal productivity of labour in the manufacturing sector. This simplification is deemed reasonable as the focus of the analysis done with the model is the fisheries market and changes in harvesting technology. Production of the numeraire commodity available to consumers is then determined as total production given the labour available left over after subtracting that used for economy wide fixed costs. Labour available to production is given by the difference between labour employed in the harvesting industry and total labour provided by households, normalized to unity.

$$y = \omega \left(1 - \sum_{k=1}^{\bar{k}} n_k e_k \right) - \sum_{k=1}^{\bar{k}} n_k \phi_k \tag{19}$$

With this the model is fully defined. The solutions to the market equilibrium for fish are presented below for an open-access and optimal demand setting. The open-access solution to the market equilibrium stemming from market interaction between harvesters and households was derived by Blanz (2018) and is reproduced below in order to be compared to the socially optimal solution derived in this paper.

2.5 Open-Access

Under open-access harvest rates are determined through the market equilibrium between utility maximizing households and profit maximizing firms. The market is assumed to have perfect competition driving profits of individual harvesters to zero. Harvest rates in the free market equilibrium only depend on the current state of the ecosystem.

Ecosystem dynamics and interactions between species are irrelevant. Interactions in harvesting and demand side interactions, however, do have an impact on the market equilibrium. Technological interactions determine the feasible set of harvesting rates described in Section 2.2. The solution to the open access market equilibrium then depends on the case of the demand function, appropriate to the edges of the harvesting set or within it. In the first case the market equilibrium lies within the space spanned by the harvesting vectors, the shaded area in Figure 2, in the second it lies on the borders. The first case is characterised by both gear types being in use. The overall composition of harvests can be changed freely. Within this case, technological interactions may be irrelevant under certain conditions. The second case implies that only one gear type is in use, dictating the proportion of the species available to the market. .

The market equlibria for both cases under open-acces presented below are derived in Blanz (2018) and reproduced here to enable comparison with the solutions under socially optimal demand. The steps of the derivation are reproduced in Appendix B. A result from that paper used when analysing the importance of the different types of interaction in the following is the condition for no effect of bycatch (23). When these conditions are met, the properties of technological interactions have not effect on prices, harvest rates or stocks. These conditions are used in the sections discussing optimal management. The open-access market solutions are used to evaluate the different management measures by analysing their impact on the open-access market equilibria. They are further used to determine incentives for deviation from prescribed harvesting rates by the economic actors.

The two cases of the market equilibrium depending on harvesting rates being limited by the feasible set or not are presented below. In the first case harvesting composition is not limited by technological interaction. In the second case proportions of species in harvests are limited to that in the catch of a specific gear type.

Case 1: All Gear Types in Use

As there are assumed to be no distortions on the market for harvested fish, fishers may freely enter or leave the market. This causes profits of fishers to be driven towards zero. Prices are then equal to the minimum average harvesting costs per species. Harvesting by individual firms is performed at the zero profit, profit maximal effort levels. Average harvesting costs are determined over all gear types available that harvest a specific species.

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \chi^{-1} \left(\boldsymbol{\nu}^{\mathsf{T}}\right)^{-1} \begin{pmatrix} \phi_1 \left(1+\frac{\varepsilon}{1-\varepsilon}\right) \left(\frac{\phi_1}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon} \\ \phi_2 \left(1+\frac{\varepsilon}{1-\varepsilon}\right) \left(\frac{\phi_2}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon} \end{pmatrix} \tag{20}$$

Market prices depend on the properties of all gear types available to the harvesters. The derivation of the minimum harvest cost equation with free harvesting combinations between species (20) is reproduced in Appendix B.2.

The Fleet composition using each of the tools is determined by balancing supply and demand, satisfying the goods market clearing condition (10).

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \left(\boldsymbol{\nu}\ \mathrm{diag}(\vec{e^{*\varepsilon}})\right)^{-1} \chi^{-1}\ \vec{q}(\vec{p}) \tag{21}$$

Quantities demanded of each of the species depending on prices $\vec{q}(\vec{p})$ are defined by the household demand function for the case with free combinations of individual species (15). Hence, harvested amounts do not only depend on harvesting properties and technological interactions contained therein but also on preferences of households and demand side interactions between species. In this case, where households may freely choose the relative amounts of species consumed, changes in prices will consequently lead to shifts in the demanded quantities of each of the species. The mechanism behind this adjusting behaviour of demand can be seen in Figure 3. In that figure a simultaneous market equilibrium on the markets for both species is shown. The supply and demand functions for the first and second species, depending on prices of both species, are shown in the front and read section of the figure respectively. The current market equilibria for each of the species depending on only their respective prices are shown in sub panels (a) and (b). These are obtained by slicing the multidimensional markets at the levels of the prices for the respective other species, indicated by the dashed lines.

An increase in the price for species 1, due to changed ecosystem stocks, changes in harvesting parameters or management measures, shifts up the corresponding price plane. This upward shift is reflected by an upward shift in the supply curve in sub panel (a). However, simultaneously, the increase in the price of species moves the market equilibrium for species 2 along the intersection of the supply and demand planes on that market. This movement along

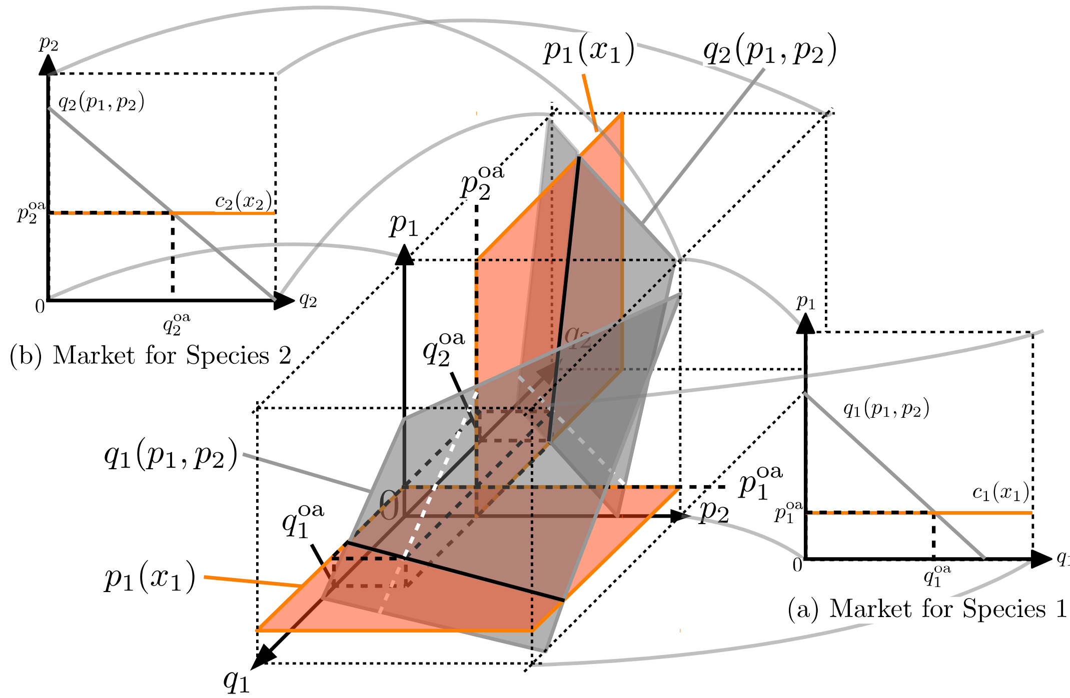

Figure 3: Sketch of simultaneous market equilibria for both fish species when both gear types are in use. The market equilibria for the individual species are shown in sub-panels (a) and (b) for Species 1 and 2 respectively. Market demand functions depend on the price of both species. The market supply planes only depend on the stock of their respective species. The bold lines along the intersections of the supply and demand planes indicate possible market equilibria for each of the two markets. The global equilibrium is found where the these curves, projected into either the price space (p_1, p_2) or the quantity space (q_1, q_2), intersect. The feasible set of combinations for q_1 and q_2 derived in 2.2 cannot be shown in this representation.

the functions would be mirrored by an upward shift of the demand function in sub panel (b). This simple example explains why managing different species as if the demanded quantities where independent, may lead to unexpected results, even in cases where harvesting costs are independent of each other.

Regarding technological interactions in harvesting, it would appear that any difference in parametrisation of catch efficiencies would also influence harvesting costs, prices, quantities demanded and finally stocks. But this is not necessarily the case. As is shown below, under certain conditions different parametrisations of catch efficiencies may yield identical results.

Conditions for no Effect of Technological Interaction

Only when both gear types are in use is it possible that changes in fleet composition perfectly offset differences in harvesting efficiency parametrisations.

$$n_k > 0 \,\forall k \in K \tag{22}$$

Furthermore costs of the gear types under consideration need to satisfy equation (23) with regards to their relative expensiveness and relative total harvesting efficiencies of both tools, conditioned on the returns to effort in harvesting.

$$\frac{\phi_1}{\phi_2} = \left(\frac{\nu_{11} + \nu_{21}}{\nu_{12} + \nu_{22}}\right)^{\frac{1}{1-\varepsilon}} \tag{23}$$

If bycatch intensity is interpreted as a change in species composition of harvests with a given harvesting gear, but not a change in total harvests using that gear, the sum of the individual harvesting efficiencies of each of the gears will remain constant $v_{1k} + v_{2k} = \text{const}$. The derivation of these conditions first shown by Blanz (2018), is reproduced in Appendix B.6 for the reader's convenience.

If these conditions are met, technological interactions in harvesting can be ignored in modelling. This condition together with the further restriction that ecosystem interactions may not be present also applies to the optimal management solution derived in Section 2.6.

Case 2: One Gear Type in Use

When the quantities of market equilibrium are limited to multiples of one of the harvesting vectors, the quantities demanded by consumers determining the market equilibrium are derived as follows: The size of the fishing fleet, using the single gear, is determined by the single tool demand function (17) depending on harvesting costs and catch efficiency. The corresponding quantities are determined by the total harvesting equation (5) combining fleet size and per vessel harvests. Prices are determined according to the inverse demand function (16) depending on the quantities. The three relevant equations are repeated below for the benefit of the reader.

$$n_k = c_k^{-\eta} \alpha^{\eta} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \tag{17}$$

$$q_i = H_i = \sum_{k=1}^{\bar{k}} n_k h_{ik}(e_k, x_i) \tag{5}$$

$$p_i = \alpha q_i^{-\frac{1}{\sigma}} Q^{\frac{\eta-1}{\eta} - \frac{\sigma-1}{\sigma}} \tag{16}$$

2.6 Optimal Harvest

To determine the socially optimal harvesting rates for all species, taking into account all three types of potential interaction between species, an all knowing benevolent social planner or manager is assumed. The aim of this manager is to maximize household utility not only for the current period but also for all future periods. The relative importance between utility today and in the future is given by the social discount rate ${}^t\rho$. In order to maximize inter-temporal household utility, harvest rates are set such that ecosystem stocks grow to the optimal amount with regards to possible sustainable harvests. Once these stock levels are reached the system will be kept in a steady state, where harvests equal stock growth in each period. This long term optimal steady state is determined below. While the optimal dynamics leading into the steady state appear to intractable, derivatives of the steady state with respect to model parameters or state variables can readily be analysed.

Comparing the harvest rates determined for the steady state under optimal access it is immediately obvious that they will be lower than those under open-access given identical stock levels. However, the higher open-access harvesting rates would deplete stocks. The open-access steady state will then have lower harvesting rates than those under optimal management. The differences between the market results under open-access and optimal harvesting for species are shown qualitatively in Figure 4. The optimal demand function, incorporating the shadow prices of stock depletion μ_1 is shifted downward from that under open-access. Alternatively, shadow prices can be thought of as a mark-up on market prices p_1^{oa}, evaluated by the short term demand function. Lower demand combined with unchanged harvesting costs result in the lower optimal harvesting rate q_1^*. However, due to only incorporating the market for one of the species, this figure belies the additional complexity in the analysis stemming from interactions between species. It is included here to aid the intuition of the reader. It should be understood in context with Figure 3. Hence, a shift in the demand curve is actually a shift of the demand plane, depending on both prices, occurring simultaneously in both markets.

The goal of the all knowing benevolent manager is measured by welfare W. This includes harvests in all periods of the model. Hence, the derived socially optimal harvest rates do not only depend on the current state of the ecosystem, as they do in the open-access case. Instead they also depend on the changes of stocks over time. Therefore, determining optimal harvest rates in this way necessarily takes into account all three avenues of interaction between species. Ecosystem interactions are included through the time derivatives of stocks. They are needed to correctly incorporate the ability of the ecosystem stock to recover from human impacts through harvesting. Technological interactions determine how species are harvested, as in the open-access case. Demand side interaction between species are given by the household preferences.

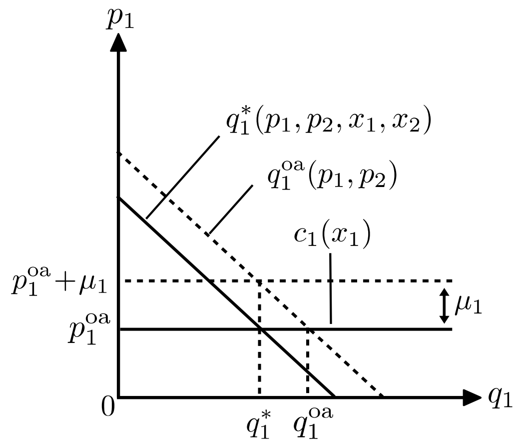

Figure 4: Market equilibrium for Species 1 with socially optimal demand and shadow prices.

$$W = \max_{\vec{q},y} \int_0^T U(\vec{q},y)\ \mathrm{e}^{{}^t\rho t}\ \mathrm{d}t \tag{24}$$

Formally the optimal harvesting decision maker maximizes the time integral over the household utility function continuously discounted with discount rate ${}^t\rho$ subject to stock change (2) and per period budget constraints of households (14). In order to do this the decision maker chooses consumption levels of each of the species q_i and levels of other consumption y. Due to tractability issues of the model the socially optimised harvest rates are only determined for the steady states of the ecosystem, where harvests are exactly equal to the intrinsic growth of ecosystem stocks.

As in the open-access market equilibrium, technological interactions in harvesting limit the possible proportions of species in harvests and consumption. This limit stems from the supply side of the market being restricted by the the zero lower bound on fleet sizes (11). As a consequence the optimal equilibrium of the fish market has two cases depending on whether that condition is binding. In one case both gear types are available, and the manager may freely choose the species composition of harvests, in the other only one gear type is used and the manager is restricted to choosing the fleet size operating that gear. The resulting market equilibria for both cases are described below. The necessary derivations for each of the two cases are performed in Appendix C of this paper. The condition for switching from the first case to the second is given by (25), which is derived in Appendix C.3.

$$0 \geq \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}} \right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} (\chi_{i''} \nu_{i''2})^{\frac{\sigma-1}{\sigma}-1} \right] - \frac{\phi_1^{1-\varepsilon} \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1}]}{\phi_2^{1-\varepsilon} \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2}]} \tag{25}$$

The optimality measure described here only impacts the demand side of the model. It is assumed that harvesting firms behave as in the open-access case described above. Consequently, harvesting costs, determining prices in the first case, are equal to the open-access scenario.

Case 1: All Gear Types in Use in Optimal Harvesting

With all gear types in use, the socially optimal market equilibrium is characterised by the socially optimal demand function (26), with the possibility of freely setting species compositions in harvests. This function has an analogous structure to the household demand function shown in the previous section (15), derived in Blanz (2018). However, for each species a corresponding shadow price of stock depletion μ_i is added to the market prices p_i.

$$q_i = \alpha^{\eta} (p_i(\vec{x}) + \mu_i)^{-\sigma} \left(\sum_{i'=1}^{\bar{i}} (p_{i'}(\vec{x}) + \mu_{i'})^{1-\sigma} \right)^{\frac{\sigma-\eta}{1-\sigma}} \tag{26}$$

The shadow prices on stock depletion μ_i internalise the externalities caused by harvesting ecosystem stocks that are not included in the open-access scenario. They are determined depending on the stock growth equations $g_i(\vec{x})$

and the Jacobian of stock growth $\boldsymbol{J}(\vec{g}(\vec{x}))$ as well as price derivatives. The presence of the growth functions ensures that impacts from harvesting on the future prospects of each of the species is taken into account, while the cross derivatives are necessary to incorporate possible knock on effects from ecosystem interactions between species. The price derivatives incorporate changes in harvesting costs in the future due to current harvesting.

$$\vec{\mu} = \left({}^{t}\rho \boldsymbol{I}^{\bar{i}} - (\boldsymbol{J}(\vec{g}(\vec{x})))^{\intercal}\right)^{-1} \begin{pmatrix} -\frac{\partial p_1(\vec{x})}{\partial x_1} g_1(\vec{x}) \\ -\frac{\partial p_2(\vec{x})}{\partial x_2} g_2(\vec{x}) \end{pmatrix} \tag{27}$$

Prices (20) and fleet sizes (21) are determined identically to the open-access mode of the model. This is the case as inter-temporal consideration by the demand side of the market do not impact firm decision making or cost structures.

Regarding the importance of the different types of interaction between species, it is obvious from comparing the socially optimal solution for this case with that under open-access that ecosystem interactions now also need to be included. In the open-access scenario, they could safely be omitted. Technological interactions meanwhile again appear to play a significant role and therefore should not be omitted when determining optimal harvest rates. However, as in the scenario under open-access, for which conditions are given in Blanz (2018) and reproduced in the section above, conditions can be determined when such interaction or changes thereof do not change results. These are derived below. Demand side interactions meanwhile are present unconditionally.

Conditions for no Effect of Technological Interaction under Optimal Harvesting

In Blanz (2018) a chain of arguments regarding the simultaneous pressure on prices from adjusting the relative catch amounts of harvesting gear is used to motivate the derivation of the "conditions for no effect of bycatch". It remains to be shown that these conditions also hold for the setting with optimal harvesting rates. The starting point for the analogous argument chain is the welfare maximizing demand function for the case where both types of gear are in use (26). This function depends on shadow prices of stock depletion defined by (27) and prices. As a market equilibrium is assumed, prices are determined by harvesting costs (20). Given that harvesting costs are determined independently of the optimal harvesting rates, these are not different from the open-access case. Hence, the condition under which prices are independent of technological interaction (23) holds without change. Shadow prices depend on the direct derivatives of prices with respect to corresponding stocks multiplied by the inverse of the Jacobian of the growth function subtracted from the discount rate diagonal matrix. If the cross derivatives of the growth functions are zero, i.e. no ecosystem interaction between species is present, each shadow price only depends on the stock of a single corresponding species (28).

$$\mu_i = \left({}^{t}\rho - \frac{\partial g_i(\vec{x})}{\partial x_i}\right)^{-1} \left(-\frac{\partial p_i(x_i)}{\partial x_i} g_i(\vec{x})\right) \tag{28}$$

Changing relative proportions of species in harvests using a certain gear type implies that total harvesting efficiency constant for that gear type remains constant. Consequently shadow prices μ_i will remain constant for different parametrisations of technological interaction in harvesting when the impact on shadow prices of changing relative harvesting efficiencies for one species is exactly equal to that on the other species. This is the case, as then only relative harvesting efficiencies are changed, an increase in one harvesting efficiency implies an equal decrease in the harvesting efficiency of the other species.

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}\nu_{1k}} = \frac{\mathrm{d}\mu_i}{\mathrm{d}\nu_{2k}} \quad i \in I \quad k \in K \tag{29}$$

This simplifies to the same condition under which technological interaction in harvests has no effect on prices in the open-access setting (23). Thereby it is shown that the similar conditions for no effect of technological interaction apply under open-access and optimal harvesting. The only difference is the added requirement on ecosystem interaction in the case of optimal harvesting. The steps of the simplification can be found in Appendix C.4.

Given these conditions, only when fleet composition can change freely and is of no concern to the modeller or manager, ecosystem interaction is not an issue and harvesting efficiencies and costs obey condition (23) may technological interaction between species be disregarded.

Case 2: One Gear Type Used in Optimal Harvesting

When the zero lower bound on fleet sizes (11) is binding, optimal harvesting rates are limited to determining the size of the single active fleet, analogous to the restricted open access case. In addition to the parameters and properties defining contemporary harvesting choices, optimal harvest rates also depend on the shadow prices of stock depletion μ_i.

$$n_k = \alpha^{\eta} \left(c_k + \sum_{i'=1}^{\bar{i}} \mu_{i'} h_{i'k}(x_i) \right)^{-\eta} \left(\sum_{i=1}^{\bar{i}} \left(h_{ik}(x_i) n_k \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \tag{30}$$

Shadow prices on stock depletion in this case depend on the discount rate ${}^{t}\rho$, growth equations, their derivatives and stock dependent harvesting efficiency. If ecosystem interactions between species are present, shadow prices further depend on the cross derivatives of the stock growth equations.

$$\vec{\mu} = {}^{\mu}A^{-1}\, {}^{\mu}\vec{b} \quad {}^{\mu}A(\vec{x}) \in \mathbb{R}^{\bar{i}\times\bar{i}} \quad {}^{\mu}\vec{b}(\vec{x}) \in \mathbb{R}^{\bar{i}} \quad \vec{\mu} \in \mathbb{R}^{\bar{i}} \tag{31}$$

$$\begin{aligned}
{}^{\mu}a_{ii} &= {}^{t}\rho + g_i(\vec{x}) \frac{\partial \chi_i}{\partial x_i} \chi_i^{-1} - \frac{\partial g_i(\vec{x})}{\partial x_i} \\
{}^{\mu}a_{ii'} &= -\frac{\partial g_{i'}(\vec{x})}{\partial x_i} \qquad i \neq i' \\
{}^{\mu}b_i &= \alpha \left(\sum_{i''=1}^{\bar{i}} \left(g_{i''}(\vec{x}) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \left(g_i(\vec{x}) \right)^{\frac{\sigma-1}{\sigma}} \frac{\partial \chi_i}{\partial x_i} \chi_i^{-1}
\end{aligned}$$

Prices are then determined according to the inverse household demand function (16) and quantities of individual species according to total harvesting equation (5).

3 Management Implementation

Depending on how optimal harvesting rates are defined and implemented and how well they are enforced, species interaction effects may cause actual harvesting rates to deviate from those the manager is trying to achieve. In the context of this paper, the goal of management is to achieve the optimal steady state harvest rates determined in the previous section. I investigate three methods of implementing managed harvesting rates. Two are intended to represent management measures used in reality and one is a theoretical benchmark against which effectiveness of management can be measured. The benchmark is provided by a scenario with perfect control. In the two more realistic scenarios, harvesting is influenced through Total Allowable Catch (TAC) landing quotas and quantity based taxation, respectively. Under perfect control the socially optimal demand function replaces that of the representative household, ensuring the optimal outcome. This is considered to be the baseline optimal case against which the effectiveness of the other implementations is compared, with regards to harvesting rates, stock levels, consumption levels and consumer utility. The total allowable catch regulation is modelled as limiting the quantities harvested to those determined by the manager, whereas in the taxation scenario a tax rate set by the manager to mirror social costs of harvesting is added to prices, in order to internalise the social costs of stock change in the market equilibrium. Both of these scenarios are modelled by utilizing the short term demand function derived from household preferences (15) and modifying harvest rates or prices.

For the two realistic scenarios incentives for deviation from the intended harvest rates and discarding are analysed. I will show that the main difficulties in ensuring harvest rates matching those intended by the manager are caused by technological and demand side interactions. Ecosystem interactions, if they are known and taken into account by the manager, are not an issue in the context of compliance.

In analysing the consequences of the three types of interaction discussed in this paper, I focus especially on technological interaction. To this end, I also investigate the consequences of misspecifying the degree of technological interaction in harvesting for the effectiveness of the management measures mentioned above. Misspecification includes the possibility of assuming wrong relative proportions of harvesting efficiencies or disregarding technological interaction completely.

3.1 Perfect Control

In the case of perfect control the household is replaced by the manager as the demand side of the market. All household consumption decisions are then determined in such a way as to maximize long term welfare as opposed to short term utility. This implies that it is possible to reach any desired steady state of the ecosystem through determining harvesting rates appropriately. The transition period from any initial stocks before management comes into effect can not considered here as tractability of the model does not allow determining harvest rates during this period.

Long term welfare may require harvest rates on the boundary of the feasible species compositions in harvesting, even while demand according to short term utility consideration would determine harvest rates to be strictly within that space. In the case of perfect control this is not an issue, however under the more realistic management schemes described below this may cause deviation from the intended harvest rates.

3.2 Total Allowable Catch Quotas

One of the simplest measures in order to limit harvest rates to those set by the manager is to implement total allowable catch quotas (TAC) that apply industry wide for all fishing efforts. In the context of the model this implies exogenously forcing harvested quantities to be equal to those selected by the manager. As can easily be seen from Figure 4 together with Figure 3 this would lead to the market clearing condition (10) being violated. If that is the case, prices are no longer fully determined given the quantities consumed, as minimum harvesting costs will be lower than the willingness to pay by consumers. Nonetheless, this management method is attractive to managers as it appears to have comparatively low informational requirements. Harvest rates and corresponding ecosystem impacts can seemingly be set independently of the cost structure of harvesting firms and knowledge of catch rates of individual species and the technological interaction involved. However, in order for the ecosystem impact to be limited to that allowed by the TAC, harvesters may not discard parts of their catch while at sea. This is an issue whenever harvesting is not sufficiently selective, or in the terms of this paper when technological interaction in harvesting is high, and the TAC are not within the feasible harvesting space. In this case the fishers have an incentive to change the species composition of their catch to fit that of the TAC, by discarding catch of an individual species. Thereby landings will be in compliance with the TAC, while the total impact on the ecosystem, measured in fisheries induced mortality on ecosystem stocks, will be higher.

The potential ecosystem impact caused by this behaviour can be determined from the tool specific harvesting equations (6). The first step is to determine if the TAC amounts lie within the feasible set of harvesting. Only if this is not the case, does the incentive to discard while complying with the TAC exist in the way described above.

$$\frac{h_{ik'}(x_i)}{h_{i'k'}(x_{i'})} < \frac{q_i^{\text{TAC}}}{q_{i'}^{\text{TAC}}} < \frac{h_{ik}(x_i)}{h_{i'k}(x_{i'})} \tag{32}$$

When this condition (32) does not hold, potential mortality from TAC compliant landed harvests are calculated by plugging in the effort and fleet size needed to catch one of the species at the TAC quantity into the harvesting equation for the other, thereby determining the simultaneous catch amounts of the other species. The difference between catch amounts and TAC give the amount of discarded fish.

For the following analysis let species and gear types be indexed such that species i is prescribed to be caught in a greater proportion than i' and that only gear type k would be used in harvesting

$$\frac{h_{ik}(x_i)}{h_{i'k}(x_{i'})} < \frac{q_i^{\text{TAC}}}{q_{i'}^{\text{TAC}}} \tag{33}$$

then the quantities harvested of species i' will be in accordance with the TAC $h_{i'k}^{\text{TAC}}(x_{i'}) = q_{i'}^{\text{TAC}}$, while species i will be caught in greater amounts $h_{ik}^{\text{TAC}}(x_i) > q_i^{\text{TAC}}$ and discarded as needed to meet the TAC. The derivation of the simultaneous harvest rates satisfying the TAC for one of the species are shown in Appendix D.

$$h_{ik}^{\text{TAC}}(x_i) = q_{i'}^{\text{TAC}} \frac{\nu_{ik}\chi_i}{\nu_{i'k}\chi_{i'}} \tag{34}$$

Total mortality for both species is equal to the calculated harvest rates $h_{ik}^{\text{TAC}}(x_i)$ and $h_{i'k}^{\text{TAC}}(x_{i'})$, respectively. Discards d_i of species i are given by the difference between q_i^{TAC} and $h_{ik}(x_i)$.

$$d_i = q_i^{\text{TAC}} - h_{ik}^{\text{TAC}}(x_i) \tag{35}$$

Increased mortality due to discarding can theoretically be avoided by a discard ban. With this in effect, harvesting would be shut down completely whenever the quantities caught of one of the species is equal to the TAC. Consequently harvests of the other species will be lower than that allowed by the TAC. These harvests can also be calculated using equation (34) above, by simply switching the indexing of the two species considered such that the direction of the inequality (33) is reversed. In that case harvest of the species not equal to TAC would be lower than prescribed $h_{ik}^{\mathrm{TAC}}(x_i) < q_i^{\mathrm{TAC}}$. The amount of under-harvesting of that species would then be obtained as the difference between TAC and actual harvest, analogous to the over-harvesting induced increased mortality. However, while monitoring of fleet wide discarding behaviour is possible, enforcing a discard ban is not cheap and may be prohibitively expensive (e.g. Sutinen and Andersen (1985); Daan (1997); Da Rocha et al. (2012)). Enforcement is especially costly as the action to be controlled occurs on each individual vessel while at sea. Furthermore, catch rates are stochastic and composition of individual catches may vary even without any discarding taking place, making enforcement of species composition in landings impractical.

This incentive for discarding can be avoided if TAC are set so that they are within the feasible set of harvesting rates, given the gear types available. Hence, while the attractiveness of this management measure rests in part on the idea that it requires no information about harvesting technology, the arguments in this section show that technological interaction needs to be taken into account, unless effective enforcement of a discard ban can be ensured. The other two types of interaction between species considered in this paper have no direct bearing on the effectiveness of this management measure. However, they are crucial in determining the TAC levels themselves. Disregarding ecosystem interaction in determining TAC would have obvious and well documented negative consequences for the impacted species, as described in the introduction. Whether consumer demand interactions between species matter partly depends on the goal of the manager. If the goal is simply to ensure a certain ecosystem state, the technological interaction may even yield an over-achievement of the goal, assuming lower harvests of one species due to an enforced discard ban. But even in such a case, where the goal is not to maximize welfare, consumer preferences matter as it is likely that choosing TACs that are not welfare maximizing do not provide harvested quantities in the same proportions as society demands. Furthermore, TACs may be too high or too low compared to the socially optimised harvest rates depending on the social discount rate ${}^t\rho$ and that implicitly assumed by the manager.

A benefit of TAC in comparison to tax based management, described below, is that it is comparatively robust with regards to misspecified technological interaction in harvesting, as long as the TAC are within the actual feasible set or enforcement of a discard ban can be achieved. If these conditions are met, even with a misspecified management quantities will equal the intended amounts. Under tax based management, the harvested amounts will deviate in such a situation with a high likelihood.

However, even if quantities are determined in an optimal manner and lie within the feasible set, the issue of the goods market clearing condition being violated still remains. Given that the TAC will not be equal to the market equilibrium, prices become undefined within the range between minimum harvesting costs and those appropriate to the quantities defined by the TAC. This can be seen in Figure 4 if q_1^* is taken to be the TAC. Households would be willing to pay $p_1 + \mu_1$ while harvesting costs would remain at p_1. In this case fishers may achieve positive profits by increasing prices above the minimum harvesting costs. Alternatively, consumers could reap additional surplus by keeping prices at the minimum harvesting costs. In the first case the efficiency of the market equilibrium, ensuring that only the minimum required resources are used in fishing, would be lost. The second case would would be equal to the optimal result under direct control. The final result could be either of these outcomes or a combination of both, depending on the relative market power of consumers and fishers.

3.3 Quantity Based Taxes

Taxes based on harvested quantities either in the production or the consumption of fish may either cause a downward shift in the demand curve, implying that for any given price less fish will be demanded, or an upward shift in the supply curve, implying that any quantity of fish will become more expensive. In both cases the result is the same: The equilibrium quantity of the market will decrease. Ideally, the new quantities are equal to the socially optimal harvest rates. If tax revenue is then returned to households in the form of lump sum transfer allowing additional consumption of the manufactured good, the optimal result will be achieved.

Taxes intended to shift the market equilibrium into the the social optimum from a socially suboptimal market outcome are typically called pigouvian taxes (Pigou, 1920). The socially optimal outcome is obtained within the market by internalising any market externalities. The externality stemming from fisheries is the reduction of stocks available to the future. Current harvests have an impact on social welfare in addition to the direct benefit from consumption, as welfare also includes changes in future consumption due to reduced stocks. This impact is neither included in harvesting costs nor consumer demand. The magnitude of this externality on welfare is determined as part of the derivation of the socially optimal harvest rates (Section 2.6) and is represented by the shadow prices on stock depletion $\vec{\mu}$. If harvested quantities are taxed at a rate equal to these shadow prices, tax based management will achieve the same harvested quantities as under perfect control or TAC based management with optimally set quantities. In this section I will investigate the importance of the three possible types of interactions between species for tax based management.

As in the case of management by setting total allowable catch (TAC) quantities, taking ecosystem interactions into account is necessary in order for the harvest rates intended by management to be appropriate. This is the case as shadow prices of stock depletion (27) and (31) depend on the stock growth functions of all species impacted by harvesting. Simultaneously, technological interaction needs to be taken into account, as shadow prices also depend on the changes in future harvesting costs from stock change. Demand side interactions, meanwhile, need to be taken into account when setting taxes, as they determine how consumer demand will react to the increased prices. This is especially apparent in the cases where market demand limits harvesting to one gear type. In this case the optimal tax rates also depend on the parameters governing substitution between species σ, price elasticity of fish consumption η and overall relative importance of fish in consumption α. This is ensures that the changing net prices do not change the species composition of demand. If these properties of consumer preferences are not taken into account, substitution between species in consumption may shift harvest composition into the range where both gear types are used to harvest. As a consequence harvesting costs would change, impacting prices. Depending on the strength of this price adjustment the new harvest rates may be higher than those intended by management. This market adjustment would occur only after taxes had come in effect. Hence, it would not be correctly anticipated by a manager ignoring demand side interactions.

Comparing tax based management to TAC regulation, misspecification of technological interaction in harvesting or interaction between species in consumption will have a direct effect on harvested quantities, due changes in prices and corresponding substitution behaviour by consumers caused by such a misspecification. However, where fisheries may operate at inefficient levels under TAC quotas, tax based management ensures that all harvesting is performed with average cost minimizing effort levels. This is the case, since taxes to not create a wedge between supply and demand on the market for fish, but shift one of the curves, depending on where the tax is applied, in order to move the market equilibrium to a socially desirable location. If the goal of management is to set harvest rates at the socially optimal levels, both types of management have the same information requirements. However, when the goal of management is merely to set harvest rates ensuring that exogenous ecosystem targets are met, the informational costs necessary to set taxes are far greater than those needed to set TAC for the same goal.

4 Results and Discussion: Which Types of Interaction Matter

Using the model and the market equilibria derived it is possible to give an answer to the question posed in the introduction of this paper, which of the three types of interaction between species, ecosystem interaction, technological harvesting in harvesting and demand side interaction from consumer preferences necessarily have to be considered when managing multi-species fisheries. The answer to this question is motivated by considering when omitting some or all of these interaction types will impact effectiveness of management measures in achieving their intended goals. In short, the answer for all three types of interaction is "it depends". In the case of ecosystem interactions it depends on the time horizon of the decision maker. In the case of technological interactions it depends on the presence of ecosystem interactions, harvesting properties and the goal of management. In the case of demand side interactions it depends on the goal and stringency of management. The reasoning behind each of these answers as well as a more detailed explanation of the inter-dependencies between each of these interactions between species is given in the respective sections below.

4.1 Ecosystem Interaction

Interactions between species imply that the growth of an individual species depends not only on the ability of that species to reproduce, described by its ecosystem parameters, but also on the presence of other species in the ecosystem. Regardless of the specifics of this interaction, the simple property that growth is not independent of the species under consideration implies that the cross-derivatives of the stock growth function of the individual species with respect to other species must not be zero.

$$\frac{g_i(\vec{x})}{x_i'} \neq 0 \quad i \neq i' \quad i, i' \in I \tag{4}$$

Whenever these cross-derivatives are present in the solutions of the model and condition (4) holds, ecosystem interaction needs to be taken into account when determining harvesting rates. For the optimal harvesting solution the cross derivatives are present in the shadow prices of stock depletion (27) and (31) as part of the Jacobian of the stock growth functions. Only for management with a very short time horizon, approximated by the open-access solution of the model, stock growth is not considered at all and neither are its cross derivatives. Hence, ecosystem interactions have to be taken into account for any type of management deserving of the name.

In the context of the market for fish products, harvesting induced stock changes, with or without ecosystem interaction, can be considered as externalities to the market. Stock change is directly impacted by market interactions through the fishers' production function. The direct effect from this impact is reflected in household utility through consumption, but the indirect impact on social welfare through decreased future harvesting potential is not included. This decreased future harvesting potential critically depends on the ecosystem growth functions. Hence, any management internalising the effect of stock change, needs to include all components of stock change, including ecosystem interactions. As the laissez-faire type of management, i.e. open-access, does not internalise any part of the externality, it does not depend ecosystem interactions.

4.2 Technological Interaction

The case of technological interaction in the harvesting of species is simultaneously simpler and more complex than that of ecosystem interaction. It is conceptually simpler, in the context of the market for fish products, as it is included in the production and thereby directly included in prices. It does therefore not require to be internalised by management measures in order to be present in the market equilibria. However, even though it is present irrespective of the management type, it does not necessarily have an impact on outcomes. This implies that in the cases where it does not, it can safely be ignored. Conditions for this to be the case are derived for optimal harvesting in Section 2.6.

Technological interaction is represented in the model through the elements off the diagonal of the gear harvesting efficiency matrix ν (the diagonal elements give the harvesting efficiency for the target species of each gear type). The presence of such interaction is therefore modelled by these elements being different from zero.

$$\nu_{ik} \neq 0 \quad i \neq k \quad i \in I \quad k \in K \tag{36}$$

The conditions determining when this type of interaction can be safely omitted from modelling and management consideration are fourfold and need to be satisfied simultaneously. 1. Fleet composition can change freely and is not considered by management. 2. Ecosystem interactions between species are not an issue. 3. Relative costs and harvesting efficiencies satisfy the condition determining independence of prices from changes in relative harvesting efficiencies (23). 4. total harvesting efficiency over all species using a specific gear type is not changed. The first condition is required, as in order for technological interactions in harvests to have no effect on ecosystem stocks, fleet sizes need to be able to compensate the per vessel harvest amounts. In order for this compensation to be perfect, the third condition must also be met. Furthermore, fleet sizes will not be able to adjust, if they are restricted by the zero lower bound (11). There are two possibilities for the second condition to be satisfied. Either the ecosystem does not include species interaction or management is of the laissez-faire type, implying that ecosystem interaction is ignored even if it is present. The last two conditions are related to the first, as they depend on the parameters that govern how fleet size responds to different degrees of technological interaction. If these parameters are such that minimum harvesting costs of all species do not change, (23) is satisfied, fleet sizes adjust such that total harvests do not depend on the degree of interaction between species in harvests. The reasoning why it is sufficient for prices to remain unchanged in order to safely disregard technological interaction in harvesting is given by the following chain of arguments: From the consumer demand function (15) (and its socially optimal analogue (26)) it is obvious that if prices (and shadow prices) do not change, neither does the consumed quantity of fish products. If the quantities consumed do not change, neither does total harvest, due to assumed perfect markets implying that nothing is wasted and so the goods market clearing condition (10) holds. Finally, if total harvests do not change, neither do stock levels.

Outside of these rather narrow conditions for irrelevance of technological interactions in harvests for management. The goals of management may give further reason why a modeller may choose to omit this type of interaction. If management only considered ecosystem properties and determines harvested quantities as the maximum tolerable impact on the ecosystem from harvesting, the exact harvesting properties would also appear irrelevant. However if these harvest rates are infeasible given the available harvesting gear harvesters have a strong incentive to deviate from these harvest rates. This effect is described in Section 3 above for implementation through total allowable catch quotas (TAC).

4.3 Demand Side Interaction

When the goal of management is to maximize welfare, consumer preferences are a key determinant of harvest rates. If the preferences indicate a dependence of the utility from consuming one species on the consumption level of another, ignoring this would lead to harvesting rates that do not maximize welfare. However, if the management of the ecosystem aims to achieve not maximal welfare but other goals, such as maintaining stocks above certain specific ecological thresholds, it would appear that consumer preferences may be disregarded. However, through their role in determining demand for harvested species consumer preferences have further impact regarding the compliance with management measures.

They are therefore especially important when implementing taxes. Taxes change the net prices consumers face, hence if taxes are not set appropriately, the substitution behaviour they induce may cause harvests of other species to increase disproportionately. This effect is shown by Quaas and Requate (2013) for a scenario without ecosystem or technological interaction. They demonstrate a case where an otherwise healthy and sustainably harvested species is over-fished due to management imposed on other species in the ecosystem, ignoring the substitution effects in consumption.

In the case of management implemented by total allowable catch quotas, demand side interactions between species drive incentives for fishers to discard parts of their catch increasing mortality in the ecosystem beyond that intended by the manager.

4.4 Limitations and Outlook

While the aim of the model used for the analysis in this paper is to be general, it does have a number of limitations in its applicability. The strongest limitation is that it only includes two species and gear types. While it is argued that this is sufficient in order to reflect a wide range of possible interactions between species, this is not shown conclusively. A further limitation is the omission of non-use ecosystem properties which may also be impacted by harvesting activities and relevant to welfare but are not marketed or consumed. A third limitation of the model stems from its generality. As ecosystem processes are represented quite simplistically, it is possible that further feedback effects between ecosystem growth functions and the interactions discussed exist and were mistakenly assumed to be negligible. However, this is seen as a necessary trade-off in order to keep the model tractable. Furthermore, discarding rates are not included in the fishers' choices. Hence, while the incentives for discarding stemming from setting harvesting quotas incompatible with the harvesting technology can be discussed using the model, incentives due to variance in prices can not be analysed precisely.

These limitations motivate further work in improving the model. An extension to the case with arbitrary amounts of ecosystem species and harvesting gear types will allow analysis of further ecosystem and technological interactions. This would also be the first step in incorporating non-use ecosystem services, which could provide useful results on the interactions between non-harvested and harvested ecosystem components stemming from consumer demand. The analysis of fishers incentives regarding discarding can be improved by incorporating costly discarding within the fisher's profit function. Thereby the endogenous rate of discarding, chosen by fishers given market prices, can be determined. Furthermore this will allow determining the shadow price on limited discarding, i.e. when a ban is in effect. This shadow price gives the incentive of the fisher to violate the discard ban. However, while these improvements to the model will allow derivation of new results, it is not expected that they will negate those presented above.

5 Conclusion

Interactions between different managed species, be they directly within the ecosystem, through harvesting or from consumer demand, have significant effects for the determination of optimal harvesting rules. While ecosystem interactions between species are almost trivially important in setting optimal harvesting quotas, the significance of the other types of interaction is less obvious. Depending on the goals of the manager and their specific properties they may be omitted.

Using an analytical model including all of the three types of interaction between species, ecosystem interaction, technological interaction and demand side interaction, I derived socially optimal harvest rates in order to show their dependence on each of these three types of interaction. I further investigated total allowable catch quotas (TAC) and quantity based taxes as management measures to enforce the socially optimal harvesting rates under market demand and supply derived for the open-access setting. Regarding the derived socially optimal harvesting rates, I found that ecosystem interactions can not be omitted when setting harvest rates, technological interactions in harvesting may be omitted in specific cases and demand side interactions may not be omitted. For the different management implementations of harvesting rates I found that their effectiveness strongly depends on technological and consumer demand interactions. In the case of TAC regulation technological interaction in harvesting was found to be especially important in determining the incentives of harvesters to discard parts of their catch, increasing mortality beyond that intended by management. For implementation of successful tax based management, meanwhile, knowledge of consumer substitution behaviour due to changes in net prices was shown to be essential. As a consequence, even in cases where one might assume that it is not necessary to include technological or demand side interactions, as the goal of management is not to maximize societal welfare but simply to ensure certain levels of ecosystem stocks, omitting these interactions can undermine the effectiveness of management measures.

In light of these results it appears advisable to include all three types of interactions between managed species when modelling multi-species fisheries in order to better anticipate what would otherwise be classified as unforeseen behaviour of the human actors in the system.

Appendices

Appendices A and B determining the open-access behaviour of the model are reproductions from Blanz (2018) with some modifications, shown here in order to aid comparison of the open-access and socially optimal solutions of the model.

A The Firm Optimisation Problem

In the following, vector notation is used to simplify the equations. For this the following definition is needed.

$$\chi = \begin{pmatrix} \ddots & & 0 \\ & \chi_i & \\ 0 & & \ddots \end{pmatrix} \tag{37}$$

Furthermore as above the following abbreviation is used.

$$\chi_i = \chi(x_i) \tag{38}$$

The effort level is determined by maximizing profits.

$$e_k^{**} = \arg \max_{e_k} \vec{P}^\intercal \chi \vec{v}_k e_k^\varepsilon - \omega e_k - \phi_k \tag{39}$$

The resulting first order condition is:

$$\varepsilon \vec{P}^\intercal \chi \vec{v}_k e_k^{\varepsilon-1} - \omega = 0 \tag{40}$$

Rearranging yields the optimal harvesting effort level per tool.

$$e_k^{**} = \left(\frac{\varepsilon \vec{P}^\intercal \chi \vec{v}_k}{\omega} \right)^{\frac{1}{1-\varepsilon}} \tag{41}$$

As perfect markets are assumed in the model, market pressure will drive profits to zero. These market processes are not observable in the model's results, as they are assumed to happen instantaneously and only the resulting market equilibrium is calculated.

The zero profit condition reads:

$$\vec{P}^\intercal \chi \vec{v}_k e_k^\varepsilon - \omega e_k = \phi_k \tag{42}$$

Rearranging of (41) and substituting into a rearranged (42) yields the optimal market equilibrium effort level. Note that this does not imply an equilibrium in the ecosystem stock change, but merely an equilibrium in market entry and exit of harvesting firms.

$$\begin{aligned}
(41) \Leftrightarrow && e_k^{1-\varepsilon} &= \frac{\varepsilon \vec{p}^\intercal \left(\vec{h}_k \chi \right)}{\omega} \\
(42) \Leftrightarrow && \frac{\varepsilon \vec{p}^\intercal \left(\vec{h}_k \chi \right)}{\omega} e_k^\varepsilon - e_k \varepsilon &= \frac{\phi_k}{\omega} \varepsilon \\
\Leftrightarrow && e_k (1-\varepsilon) &= \frac{\phi_k}{\omega} \varepsilon \\
\Leftrightarrow && e_k^* &= \frac{\phi_k}{\omega} \frac{\varepsilon}{1-\varepsilon} && (43)
\end{aligned}$$

Optimal market-equilibrium effort decreases with wages. This is to be expected, as increasing wages imply that the firm can not afford as much labour. The model does not allow for substitution into variable capital. There is only fixed capital, covered by fixed costs. Furthermore, increasing fixed costs increases equilibrium effort level. This can be explained by increased fixed costs implying that more effort is needed to reach break-even. This ignores the possibility of not producing using a certain technology, however.

B The Household Optimisation Problem and Market Equilibria

B.1 All Tools In Use

In the case where no non–negativity conditions are binding, the household optimisation problem reads as follows:

$$\max_{Q,y} U(Q,y) \text{ s.t. } \omega = y + \sum_{i=1}^{\bar{i}} p_i q_i \tag{44}$$

The corresponding Lagrangian function:

$$\mathscr{L}(Q,y) = U(Q,y) - \lambda(\omega - y - \sum_{i=1}^{\bar{i}} p_i q_i) \tag{45}$$

The resulting first order conditions:

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}y} = 1 + \lambda = 0 \tag{46}$$

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}q_{i'}} = \alpha q_{i'}^{\frac{\sigma-1}{\sigma}-1} \left(\sum_{i=1}^{\bar{i}} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} + \lambda p_{i'} = 0 \tag{47}$$

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\lambda} = \omega - y - \sum_{i=1}^{\bar{i}} p_i q_i = 0 \tag{48}$$

From (46) it trivially follows that $\lambda = -1$. This can be used in (47), which can then be rearranged to find the demand function for each harvested species as follows.

$$(47) \Leftrightarrow \quad \alpha q_{i'}^{\frac{\sigma-1}{\sigma}-1} \left(\sum_{i=1}^{\bar{i}} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} = p_{i'}$$

$$\Leftrightarrow \quad \alpha q_{i'}^{\frac{\sigma-1}{\sigma}-1} Q^{\frac{\eta-1}{\eta}-\frac{\sigma-1}{\sigma}} = p_{i'} \tag{49}$$

$$\Leftrightarrow \quad \alpha q_{i'}^{-\frac{1}{\sigma}} Q^{\frac{\eta-1}{\eta}-\frac{\sigma-1}{\sigma}} = p_{i'} \tag{50}$$

$$\Leftrightarrow \quad \left(\alpha Q^{\frac{\eta-1}{\eta}-\frac{\sigma-1}{\sigma}} \right)^{1-\sigma} q_{i'}^{\frac{\sigma-1}{\sigma}} = p_{i'}^{1-\sigma}$$

$$\Leftrightarrow \quad \left(\alpha Q^{\frac{\eta-1}{\eta}-\frac{\sigma-1}{\sigma}} \right)^{1-\sigma} \underbrace{\sum_{i'=1}^{\bar{i}} q_{i'}^{\frac{\sigma-1}{\sigma}}}_{Q^{\frac{\sigma-1}{\sigma}}} = \sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma}$$

$$\Leftrightarrow \quad \alpha^{1-\sigma} Q^{(1-\sigma)\left(\frac{1}{\eta}\right)} = \sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma}$$

$$\Leftrightarrow \quad \left(\alpha Q^{\frac{1}{\eta}} \right)^{1-\sigma} = \sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma}$$

$$\Leftrightarrow \quad \alpha Q^{\frac{1}{\eta}} = \underbrace{\left(\sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}}_{P}$$

$$\Leftrightarrow \quad Q = \alpha^{\eta} P^{-\eta} \tag{51}$$

Hereby a price index P is used.

$$P = \left(\sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{52}$$

(51) can itself be substituted into (49) to finally obtain the demand function (53).

$$\begin{aligned}
(49) \Leftrightarrow \quad & \alpha q_i^{\frac{1}{\sigma}} \left(\alpha^\eta P^{-\eta} \right)^{\frac{\eta-1}{\eta} - \frac{\sigma-1}{\sigma}} = p_i \\
\Leftrightarrow \quad & q_i = \alpha^\eta p_i^{-\sigma} P^{\sigma-\eta} \\
\Leftrightarrow \quad & q_i = \alpha^\eta p_i^{-\sigma} \left(\sum_{i'=1}^{\bar{i}} p_{i'}^{1-\sigma} \right)^{\frac{\sigma-\eta}{1-\sigma}} && (53)
\end{aligned}$$

B.2 Prices, All Tools In Use

Given the assumption that $\bar{i} = \bar{k}$ and the zero profit condition, prices are equal to unit costs in production. These are determined by solving the zero profit conditions of all $\bar{k}$ tools simultaneously.

First the optimal effort level satisfying the individual zero profit conditions (43) is plugged back into the respective zero profit conditions.

$$\begin{aligned}
& \underbrace{\vec{p}\,'\chi\vec{\nu}_k \Big(\overbrace{\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}}^{e_k^*} \Big)^{\varepsilon}}_{\text{revenue}} = \underbrace{\omega \overbrace{\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}}^{e_k^*}}_{\text{wage costs}} + \underbrace{\phi_k}_{\text{fixed costs}} \\
\Leftrightarrow \quad & \vec{p}\,'\chi\vec{\nu}_k \left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon} \right)^{\varepsilon} = \phi_k \left(1 + \frac{\varepsilon}{1-\varepsilon} \right) \\
\Leftrightarrow \quad & \vec{p}\,'\chi\vec{\nu}_k = \phi_k \left(1 + \frac{\varepsilon}{1-\varepsilon} \right) \left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon} \right)^{-\varepsilon} && (54)
\end{aligned}$$

Equation (54) above represents a system of $\bar{k}$ linear equations. This can be used to determine the $\bar{i}$ prices p_i. However, this requires that $\bar{k} = \bar{i}$. Writing out the vector multiplications makes the linearity more easily apparent.

$$(54) \Leftrightarrow \sum_{i=1}^{\bar{i}} p_i \chi_i(\vec{x}) \nu_{ik} = \underbrace{\phi_k \left(1 + \frac{\varepsilon}{1-\varepsilon} \right) \left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon} \right)^{-\varepsilon}}_{{}^p b_k} \tag{55}$$

To solve the system, it is rearranged as follows:

$${}^pA\ \vec{p} = {}^p\vec{b} \tag{56}$$

The top left index is used to denote that this LSE (linear system of equations) is used to solve for prices, as opposed to the LSE used to determine the number of firms below.

$${}^pA = \nu^\intercal \chi \tag{57}$$

$${}^p\vec{b} = \begin{pmatrix} {}^p b_1 \\ \vdots \\ {}^p b_{\bar{k}} \end{pmatrix} \tag{58}$$

$${}^p b_k = \phi_k \left(1 + \frac{\varepsilon}{1-\varepsilon} \right) \left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon} \right)^{-\varepsilon} \tag{59}$$

Prices can then be easily solved for.

$$\begin{aligned}\vec{p} &= {}^{p}A^{-1}\,{}^{p}\vec{b} \\ &= \chi^{-1}(\nu^{\intercal})^{-1}\,{}^{p}\vec{b}\end{aligned} \tag{60}$$

Thereby prices are fully determined.

B.3 Number of Firms, All Tools in Use

Starting from the goods market clearing conditions for each of the species the problem of determining the numbers of firms can be transformed into a system of linear equations and solved as such.

$$\begin{aligned} & H_i = q_i(\vec{p}) \\ \Leftrightarrow \quad & \sum_{k=1}^{\bar{k}} n_k \chi(x_i) \nu_{ik} e_k^{*\varepsilon} = q_i(\vec{p}) \\ \Leftrightarrow \quad & \frac{q_i(\vec{p})}{\chi(x_i)} = \sum_{k=1}^{\bar{k}} n_k \nu_{ik} e_k^{*\varepsilon} \end{aligned} \tag{61}$$

Represented as a LSE:

$${}^{n}A\,\vec{n} = {}^{n}\vec{b} \tag{62}$$

with

$${}^{n}A = \nu \;\operatorname{diag}(\vec{e^{*\varepsilon}})$$

and

$$\begin{aligned} & {}^{n}\vec{b} = \chi^{-1}(\vec{x})\;\vec{q}(\vec{p}) \\ \Leftrightarrow \quad & {}^{n}b_i = \chi^{-1}(x_i)\;q_i(\vec{p}) \end{aligned}$$

.

Solving for n yields:

$$\vec{n} = {}^{n}A^{-1}\;{}^{n}\vec{b} \tag{63}$$

B.4 One Tool in Use

In order to solve the household optimisation problem in this context, the Kuhn-Tucker conditions are used (Kuhn, 2014). To simplify the analysis the goods market clearing condition (10) is substituted into the sub-utility for fish (13) and the budget restriction is reformulated.

$$\tilde{Q} = \tilde{Q}(\vec{n}) = \left(\sum_{i=1}^{\bar{i}} \underbrace{\left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \right)}_{q_i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{64}$$

Hereby the c_k represents the operating costs of each firm of type k. The firm costs do not depend on any variables as it is assumed that firms operate at the zero-profit profit-maximizing level (9).

$$\begin{aligned} c_k &= e_k^*\omega + \phi_k \\ &= \phi_k \left(1 + \frac{\varepsilon}{1-\varepsilon}\right) \end{aligned} \tag{65}$$

Given that firms operate at the equilibrium level, the sum of costs multiplied with the number of firms must equal the sum of prices multiplied by consumed amounts of species.

$$\sum_{k=1}^{\bar{k}} c_k n_k = \sum_{i=1}^{\bar{i}} p_i q_i \tag{66}$$

This is substituted into (14) to yield the reformulated budget constraint.

$$y = \omega - \sum_{k=1}^{\bar{k}} c_k n_k \tag{67}$$

The representative household now chooses the type and number of bundles of fish in order to maximize utility. Bundles consist of certain amounts of each harvestable species. The composition of the bundles is defined by equilibrium output of a single firm of each type $h_k(\vec{x})$.

$$h_k(\vec{x}) = \begin{pmatrix} h_{1k}(x_1) \\ \vdots \\ h_{\bar{i}k}(x_{\bar{i}}) \end{pmatrix} \tag{68}$$

The Lagrangian of the household optimisation problem then is:

$$\mathscr{L}(\vec{n}) = \omega - \sum_{k=1}^{\bar{k}} c_k n_k + \alpha \frac{\eta}{\eta - 1} \left(\sum_{i=1}^{\bar{i}} \left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} - \sum_{k=1}^{\bar{k}} \lambda_k(-n_k) \tag{69}$$

As $\ln(Q)$ is the continuous extension of $\frac{\eta}{\eta-1} Q^{\frac{\eta-1}{\eta}}$ the first order conditions derived using the above equation also extend to the case $\eta = 1$.

The first order conditions are:

$$\frac{\mathrm{d}\mathscr{L}(\vec{n})}{\mathrm{d}n_k} = 0 \tag{70}$$

$$\lambda_k \geq 0 \tag{71}$$

$$-n_k \leq 0 \tag{72}$$

$$-\lambda_k n_k = 0 \tag{73}$$

To determine the solution to the household optimisation problem, it is split into cases depending on the number of tool types in use. The cases considered are:

1. All tool types are in use
2. Only one tool type is in use
3. Not all tools are in use, but more than one is in use

In order to keep the analysis simple, for the remainder of this paper only two species and harvesting tools are considered. This removes the third case from consideration.

Assumption 1. $\bar{k} = \bar{i} = 2$

The demand for the bundle provided by the active firm is derived by using the fact that the number of active firms for all other tool types is zero, in the optimality conditions of the household optimisation problem. Let there exist a single tool type with positive active firms ($n_k > 0$) and let all other tool types have zero active firms ($n'_k = 0 \ \forall k \in \{[1,\bar{k}] \setminus k'\}$). From (73) it follows that $\lambda_k = 0$.

$$(70) \Leftrightarrow \qquad 0 = \frac{\mathrm{d}\mathscr{L}(\vec{n}}{\mathrm{d}n_k}$$

$$\Leftrightarrow \qquad 0 = \lambda_k - c_k + \alpha \left(\sum_{i=1}^{\bar{i}} \left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i=1}^{\bar{i}} h_{ik}(x_i) \left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \right)^{\frac{\sigma-1}{\sigma}-1}$$

using $n'_k = 0$ and $\lambda_k = 0$

$$\Leftrightarrow \qquad 0 = 0 - c_k + \alpha \left(\sum_{i=1}^{\bar{i}} \big(n_k h_{ik}(x_i)\big)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i=1}^{\bar{i}} h_{ik}(x_i) \big(n_k h_{ik}(x_i)\big)^{\frac{\sigma-1}{\sigma}-1}$$

$$\Leftrightarrow \qquad 0 = -c_k + \alpha \left(\sum_{i=1}^{\bar{i}} \big(n_k h_{ik}(x_i)\big)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} n_k^{\frac{\sigma-1}{\sigma}-1}$$

$$\Leftrightarrow \qquad 0 = -c_k + \alpha \left(\sum_{i=1}^{\bar{i}} n_k^{\frac{\sigma-1}{\sigma}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} n_k^{\frac{\sigma-1}{\sigma}-1} \sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow \qquad 0 = -c_k + \alpha n_k^{\frac{\sigma-1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1\right)} n_k^{\frac{\sigma-1}{\sigma}-1} \frac{\eta}{\eta-1} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow \qquad 0 = -c_k + \alpha n_k^{-\frac{1}{\eta}} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}}$$

$$\Leftrightarrow \qquad n_k^{-\frac{1}{\eta}} = c_k \alpha^{-1} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{-\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} \tag{74}$$

$$\Leftrightarrow \qquad n_k = c_k^{-\eta} \alpha^{\eta} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \tag{75}$$

Equation (75) relates the number of bundles of type k demanded by the household. The number of goods demanded by the household follows from the goods market clearing condition (10).

$$q_i = n_k h_{ik}(x_i)$$

$$q_i = c_k^{-\eta} \alpha^{\eta} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \nu_{ik} e_k^{*\varepsilon} \chi_i$$

$$q_i = (e_k^* \omega + \phi_k)^{-\eta} \alpha^{\eta} \left(\sum_{i=1}^{\bar{i}} h_{ik}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \nu_{ik} e_k^{*\varepsilon} \chi_i \tag{76}$$

B.5 Condition for Only One Tool in Use

In the following the condition for single tool use is derived. Without loss of generality this is done for Tool 1. Given the assumption that $\bar{i} = \bar{k} = 2$ let $n_1 = 0$ and hence $\lambda_1 > 0$ while $n_2 > 0$ and $\lambda_2 = 0$.

Starting from (70) for Tool 1:

$$0 = \lambda_1 - c_1 + \alpha\left(\sum_{i=1}^{\bar{i}}\left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)\left(\sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i)\right)^{\frac{\sigma-1}{\sigma}-1}\right]$$

Using the asssumption that $\bar{i} = \bar{k} = 2$ and assuming without loss of generality that $n_1 = 0$ and hence $\lambda_1 > 0$ while $n_2 > 0$ and $\lambda_2 = 0$.

$$\Rightarrow \qquad 0 = \lambda_1 - c_1 + \alpha\left(\sum_{i=1}^{\bar{i}}\left(n_2 h_{i2}(x_i)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)\left(n_2 h_{i2}(x_i)\right)^{\frac{\sigma-1}{\sigma}-1}\right]$$

$$\Leftrightarrow \qquad -\lambda_1 = n_2^{\frac{\sigma-1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1\right)}\alpha\left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} n_2^{\frac{\sigma-1}{\sigma}-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}-1}\right] - c_1$$

$$\Leftrightarrow \qquad -\lambda_1 = n_2^{-\frac{1}{\eta}}\alpha\left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}-1}\right] - c_1$$

The factor $n_2^{1-\eta}$ can be substituted by (74), due to Assumption 1, I.e. because Tool 2 is the only tool used.

$$\Rightarrow \qquad -\lambda_1 = \overbrace{c_2\alpha^{-1}\left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{-\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}}}^{n_2^{-\frac{1}{\eta}}}\alpha\left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}-1}\right] - c_1$$

$$\Leftrightarrow \qquad -\lambda_1 = c_2\left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[h_{i1}(x_i)h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}-1}\right] - c_1$$

Substitute (65) for c_k and equilibrium harvests obtained by plugging (9) in (6) for $h_{ik}(x_i)$.

$$\Rightarrow \qquad -\lambda_1 = (\phi_2 + \omega e_2^*)\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2} e_2^*)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i \nu_{i1} e_1^*)(\chi_i \nu_{i2} e_2^*)^{\frac{\sigma-1}{\sigma}-1}\right] - (\phi_1 + \omega e_1^*)$$

$$\Leftrightarrow \qquad -\lambda_1 = (\phi_2 + \omega e_2^*)\, e_2^{*\frac{1-\sigma}{\sigma}}\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1} e_1^* e_2^{*\frac{\sigma-1}{\sigma}-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right] - \phi_1 - \omega e_1^*$$

$$\Leftrightarrow \qquad -\lambda_1 = e_1^*\left(\frac{\phi_2}{e_2^*} + \omega\right)\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right] - \phi_1 - \omega e_1^*$$

Substitute (9) for e_k^*.

$$\Rightarrow \qquad -\lambda_1 = \left(\frac{\phi 1}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{\phi_2\omega}{\phi_2}\frac{1-\varepsilon}{\varepsilon}+\omega\right)\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i\nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right]-\phi_1\left(1+\frac{\varepsilon}{1-\varepsilon}\right)$$

$$\Leftrightarrow \qquad -\lambda_1 = \underbrace{\phi_1\left(1+\frac{\varepsilon}{1-\varepsilon}\right)}_{c_1}\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i\nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right]\underbrace{-\phi_1\left(1+\frac{\varepsilon}{1-\varepsilon}\right)}_{-c_1}$$

$$\Leftrightarrow \qquad -\lambda_1 = c_1\left(\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\sum_{i=1}^{\bar{i}}\left[(\chi_i\nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right]-1\right)$$

$$\Leftrightarrow \qquad -\lambda_1 = c_1\left(\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\left(\sum_{i=1}^{\bar{i}}\left[(\chi_i\nu_{i2})^{\frac{\sigma-1}{\sigma}}\frac{\nu_{i1}}{\nu_{i2}}\right]-\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)\right)$$

$$\Leftrightarrow \qquad -\lambda_1 = c_1\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\left(\sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\left(\frac{\nu_{i1}}{\nu_{i2}}-1\right)\right) \tag{77}$$

From (73) in conjunction with (77) it follows that

$$0 \geq \sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\left(\frac{\nu_{i1}}{\nu_{i2}}-1\right) \tag{78}$$

where the equality holds only if (72) is not binding. Conversely the condition for Tool 1 not being employed is:

$$0 > \sum_{i=1}^{\bar{i}}(\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\left(\frac{\nu_{i1}}{\nu_{i2}}-1\right) \tag{79}$$

B.6 Conditions For No Effect of Bycatch

Given assumption $\bar{i} = \bar{k} = 2$ (two species, two tools) the equation determining the prices (60) can be written, for the price of Species 1, in expanded form as

$$p_1(\nu) = (\chi_1\chi_2)^{-1}(\nu_{11}\nu_{22}-\nu_{12}\nu_{21})^{-1}(\chi_2\nu_{22}\,{}^p b_1 - \chi_2\nu_{21}\,{}^p b_2) \tag{80}$$

As it is only relevant whether the price changes, not how it changes, and as prices are directly related through the demand function, this derivation can without loss of generality be done only for the price of Species 1.

It is assumed that the sum of the total harvesting efficiency of the tools remains constant while adding bycatch to the model. For the price to remain constant the derivative of the price with respect to the increasing harvesting efficiency must equal the derivative with respect to the decreasing harvesting efficiency. Without loss of generality, let ν_{12} be the increasing and ν_{22} the decreasing harvesting efficiency.

$$\frac{\mathrm{d}p_1}{\mathrm{d}\nu_{12}} = \frac{\mathrm{d}p_1}{\mathrm{d}\nu_{22}} \tag{81}$$

$$\frac{\nu_{21}\left({}^p b_1\nu_{22} - {}^p b_2\nu_{21}\right)}{\left(\nu_{12}\nu_{21}-\nu_{11}\nu_{21}\right)^2\chi_1} = \frac{\nu_{21}\left({}^p b_2\nu_{11} - {}^p b_1\nu_{12}\right)}{\left(\nu_{12}\nu_{21}-\nu_{11}\nu_{22}\right)^2\chi_1}$$

$${}^p b_1\nu_{22} - {}^p b_2\nu_{21} = {}^p b_2\nu_{11} - {}^p b_1\nu_{12}$$

$${}^p b_1(\nu_{22}+\nu_{12}) = {}^p b_2(\nu_{11}+\nu_{21})$$

$$\frac{{}^p b_1}{{}^p b_2} = \frac{\nu_{11}+\nu_{21}}{\nu_{12}+\nu_{22}} \tag{82}$$

substitute ${}^{p}b$ with (59)

$$\frac{\phi_1\left(1+\frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon}}{\phi_k\left(1+\frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon}} = \frac{\nu_{11}+\nu_{21}}{\nu_{12}+\nu_{22}}$$

$$\frac{\phi_1^{1-\varepsilon}}{\phi_2^{1-\varepsilon}} = \frac{\nu_{11}+\nu_{21}}{\nu_{12}+\nu_{22}}$$

$$\frac{\phi_1}{\phi_2} = \left(\frac{\nu_{11}+\nu_{21}}{\nu_{12}+\nu_{22}}\right)^{\frac{1}{1-\varepsilon}} \tag{83}$$

C Intertemporal Household Optimisation Problem

The inter temporal household optimisation problem reads

$$\max_{\vec{q},y} \int_0^T U(\vec{q}_t, y_t)\, \mathrm{e}^{t\rho t}\, \mathrm{d}t \tag{84}$$

with the utility function

$$U(Q(\vec{q}_t), y_t) = \begin{cases} y_t + \alpha \dfrac{\eta}{\eta-1} Q(\vec{q}_t)^{\frac{\eta-1}{\eta}} & \text{for } \eta \neq 1 \\ y_t + \alpha \ln Q(\vec{q}_t) & \text{for } \eta = 1 \end{cases} \tag{85}$$

and the sub utility of fish consumption

$$Q(\vec{q}_t) = \left(\sum_{i=1}^{\bar{i}} q_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{86}$$

.

The optimisation is subject to the per period household budget constraint (87), the stock change equation 2, the per period goods market clearing condition (88) and the condition of non-negativity of the number of firms (89). Furthermore it is assumed that the ecosystem is in a steady state.

$$\omega = y + \sum_{i=1}^{\bar{i}} p_i q_i \tag{87}$$

$$q_i = H_i = \sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \tag{88}$$

$$n_k \geq 0 \tag{89}$$

The time index is omitted in the following.

As it is not known *ex ante* if condition (89) is binding the problem is split into cases depending on whether it is binding for each of the available tools. There are three cases to consider:

1. All tool types are in use
2. Only one tool type is in use
3. Not all tools are in use, but more than one is in use

In order to keep the analysis simple, for the remainder of this paper only two species and harvesting tools are considered (Assumption 1). This removes the third case from consideration.

C.1 Case 1

The current value hamiltonian in this case is:

$$\mathscr{H}_c = \omega - \sum_{i'=1}^{\bar{i}} p_{i'} q_{i'} + \alpha \frac{\eta}{\eta - 1} \left(\sum_{i''=1}^{\bar{i}} q_{i''}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta}} + \sum_{i=1}^{\bar{i}} \mu_i \left(r_i (x_i - \underline{x}_i) \left(1 - \frac{\vec{\gamma}_i \vec{x}}{\kappa_i}\right) - q_i \right)$$

$$\Leftrightarrow \qquad \mathscr{H}_c = \omega + \alpha \frac{\eta}{\eta - 1} \left(\sum_{i''=1}^{\bar{i}} q_{i''}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta}} + \sum_{i=1}^{\bar{i}} \mu_i \left(r_i (x_i - \underline{x}_i) \left(1 - \frac{\vec{\gamma}_i \vec{x}}{\kappa_i}\right) - q_i \right) - p_i q_i \tag{90}$$

The corresponding optimality conditions are:

$$\frac{\partial \mathscr{H}_c}{\partial q_i} = -p_i(\vec{x}) + \alpha \left(\sum_{i'=1}^{\bar{i}} q_{i'}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\eta-\sigma}{\eta(\sigma-1)}} q_i^{-\frac{1}{\sigma}} - \mu_i = 0 \tag{91}$$

$$\dot{x}_i = r_i (x_i - \underline{x}_i) \left(1 - \frac{\vec{\gamma}_i \vec{x}_t}{\kappa_i}\right) - q_i \tag{92}$$

$$\dot{\mu}_i = {}^t\rho \mu_i - \frac{\partial \mathscr{H}_c}{\partial x_i} \tag{93}$$

$$x_{i\underline{t}} = x_{i1} \tag{94}$$

$$\mu_{i\bar{t}} = 0 \tag{95}$$

These conditions are used together with the assumption that $\bar{i} = \bar{k} = 2$ to determine optimal household consumption in a steady state.

Reranging (93):

$$\dot{\mu}_i = {}^t\rho \mu_i - \frac{\partial \mathscr{H}_c}{\partial x_i}$$

$$\Leftrightarrow \qquad \dot{\mu}_i = {}^t\rho \mu_i - \sum_{i'=1}^{\bar{i}} \left[-\frac{\partial p_{i'}(\vec{x})}{\partial x_i} q_{i'} + \mu_{i'} \frac{\partial g_{i'}(\vec{x})}{\partial x_i} \right]$$

$$\Leftrightarrow \qquad \dot{\mu}_i = {}^t\rho \mu_i + \sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_i} q_{i'} \right] - \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right]$$

$$\Leftrightarrow \qquad \dot{\mu}_i = {}^t\rho \mu_i + \sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_i} q_{i'} \right] - \sum_{\substack{i''=1 \\ i'' \neq i}}^{\bar{i}} \left[\mu_{i''} \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right] - \mu_i \frac{\partial g_i(\vec{x})}{\partial x_i}$$

$$\Leftrightarrow \qquad \dot{\mu}_i - \sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_i} q_{i'} \right] = \mu_i \left({}^t\rho - \frac{\partial g_i(\vec{x})}{\partial x_i} \right) - \sum_{\substack{i''=1 \\ i'' \neq i}}^{\bar{i}} \left[\mu_{i''} \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right]$$

From the assumption of a steady state it follows that not only $\dot{\mu}_i = 0$ but also $\dot{x}_i = 0$. From the latter it follows that $q_i = g_i(\vec{x})$.

$$\Leftrightarrow \qquad -\sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_i} q_{i'} \right] = \mu_i \left({}^t\rho - \frac{\partial g_i(\vec{x})}{\partial x_i} \right) - \sum_{\substack{i''=1 \\ i'' \neq i}}^{\bar{i}} \left[\mu_{i''} \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right] \tag{96}$$

There are $\bar{i}$ of equation (96). These constitute a linear system of equations which can be written as follows:

$${}^{\mu}b = {}^{\mu}A \vec{\mu} \tag{97}$$

The components of the above are:

$$
\begin{aligned}
{}^{\mu}A &= \begin{pmatrix} {}^{\mu}a_{11} & \dots & {}^{\mu}a_{1\bar{j}} \\ \vdots & a_{ij} & \vdots \\ {}^{\mu}a_{\bar{i}1} & \dots & {}^{\mu}a_{\bar{i}\bar{j}} \end{pmatrix} \qquad a_{ii} = {}^{t}\rho - \frac{\partial g_i(\vec{x})}{\partial x_i} \qquad a_{ij} = -\frac{\partial g_j(\vec{x})}{\partial x_i} \\
\Leftrightarrow {}^{\mu}A &= -(\mathrm{J}(g(\vec{x}))^{\intercal} + {}^{t}\rho \mathbf{I}^{\bar{i}} \\
{}^{\mu}\vec{b} &= \begin{pmatrix} -\sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_1} q_{i'} \right] \\ \vdots \\ -\sum_{i'=1}^{\bar{i}} \left[\frac{\partial p_{i'}(\vec{x})}{\partial x_{\bar{i}}} q_{i'} \right] \end{pmatrix}
\end{aligned}
$$

as $\frac{\partial p_{i'}(\vec{x})}{\partial x_{\bar{i}}} q_{i'} = 0$ if $i \neq i'$, ${}^{\mu}\vec{b}$ simplifies to

$$
{}^{\mu}\vec{b} = \begin{pmatrix} -\frac{\partial p_1(\vec{x})}{\partial x_1} q_1 \\ \vdots \\ -\frac{\partial p_{\bar{i}}(\vec{x})}{\partial x_{\bar{i}}} q_{\bar{i}} \end{pmatrix}
$$

Harvested quantities q_i are equal to stock growth $g_i(\vec{x})$ as steady state conditions are investigated.

$$
{}^{\mu}\vec{b} = \begin{pmatrix} -\frac{\partial p_1(\vec{x})}{\partial x_1} g_1(\vec{x}) \\ \vdots \\ -\frac{\partial p_{\bar{i}}(\vec{x})}{\partial x_{\bar{i}}} g_{\bar{i}}(\vec{x}) \end{pmatrix}
$$

Rearranging (91):

$$
\begin{aligned}
(91) \Leftrightarrow \qquad \alpha \left(\sum_{i'=1}^{\bar{i}} q_{i'}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\eta-\sigma}{\eta(\sigma-1)}} q_i^{-\frac{1}{\sigma}} &= p_i(\vec{x}) + \mu_i \\
\Leftrightarrow \qquad \sum_{i'=1}^{\bar{i}} q_{i'}^{\frac{\sigma-1}{\sigma}} &= \left(\frac{p_i(\vec{x}) + \mu_i}{\alpha q_i^{-\frac{1}{\sigma}}} \right)^{\frac{\eta(\sigma-1)}{\eta-\sigma}}
\end{aligned}
$$

The left hand side is identical in all $\bar{i}$ cases.

$$
\begin{aligned}
\Rightarrow \qquad \left(\frac{p_i(\vec{x}) + \mu_i}{\alpha q_i^{-\frac{1}{\sigma}}} \right)^{\frac{\eta(\sigma-1)}{\eta-\sigma}} &= \left(\frac{p_{i'}(\vec{x}) + \mu_{i'}}{\alpha q_{i'}^{-\frac{1}{\sigma}}} \right)^{\frac{\eta(\sigma-1)}{\eta-\sigma}} \\
\Leftrightarrow \qquad \frac{p_i(\vec{x}) + \mu_i}{q_i^{-\frac{1}{\sigma}}} &= \frac{p_{i'}(\vec{x}) + \mu_{i'}}{q_{i'}^{-\frac{1}{\sigma}}} \\
\Leftrightarrow \qquad q_{i'} &= q_i \left(\frac{p_i(\vec{x} + \mu_i}{p_{i'}(\vec{x} + \mu_{i'}} \right)^{\sigma}
\end{aligned}
$$

plugging the above back into (91)

$$\Rightarrow \qquad p_i(\vec{x})+\mu_i = \alpha\left(\sum_{i'=1}^{\bar{i}} q_i\left(\frac{p_i(\vec{x}+\mu_i}{p_{i'}(\vec{x}+\mu_{i'}}\right)^{\sigma\frac{\sigma-1}{\sigma}}\right)^{\frac{\eta-\sigma}{\eta(\sigma-1)}} q_i^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow \qquad p_i(\vec{x})+\mu_i = \alpha q_i^{\frac{(\sigma-1)(\eta-\sigma)}{\sigma\eta(\sigma-1)}-\frac{1}{\sigma}}(p_i(\vec{x})+\mu_i)^{\frac{\sigma(\sigma-1)(\eta-\sigma)}{\sigma\eta(\sigma-1)}}\left(\sum_{i'=1}^{\bar{i}}(p_{i'}(\vec{x})+\mu_{i'})^{1-\sigma}\right)^{\frac{\eta-\sigma}{\eta(\sigma-1)}}$$

$$\Leftrightarrow \qquad q_i^{-\frac{1}{\eta}} = \alpha^{-1}(p_i(\vec{x})+\mu_i)^{\frac{\sigma}{\eta}}\left(\sum_{i'=1}^{\bar{i}}(p_{i'}(\vec{x})+\mu_{i'})^{1-\sigma}\right)^{\frac{\eta-\sigma}{\eta(\sigma-1)}}$$

$$\Leftrightarrow \qquad q_i = \alpha^{\eta}(p_i(\vec{x})+\mu_i)^{-\sigma}\left(\sum_{i'=1}^{\bar{i}}(p_{i'}(\vec{x})+\mu_{i'})^{1-\sigma}\right)^{\frac{\sigma-\eta}{1-\sigma}} \tag{98}$$

The inter temporally optimised demand function (98) is analogous to the single period demand function with added shadow prices for stock depletion.

C.2 Case 2

For the second case the problem first needs to be converted into one with bundled goods according to the production technology.

The current Value Hamiltonian in this case is:

$$\mathscr{H}_c = \omega - n_k c_k + \alpha\frac{\eta}{\eta-1}\left(\sum_{i=1}^{\bar{i}}(h_{ik}(\vec{x})n_k)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} + \sum_{1'=1}^{\bar{i}}\mu_{i'}\left(r_{i'}(x_{i'}-\underline{x}_{i'})\left(1-\frac{\vec{\gamma}_{i'}\vec{x}}{\kappa_{i'}}\right)-h_{i'k}(\vec{x})n_k\right)$$

$$\Leftrightarrow \mathscr{H}_c = \omega - n_k c_k - n_k\sum_{i''=1}^{\bar{i}}\mu_{i''}h_{i''k}(\vec{x}) + \alpha\frac{\eta}{\eta-1}\left(\sum_{i=1}^{\bar{i}}(h_{ik}(\vec{x})n_k)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} + \sum_{1'=1}^{\bar{i}}\mu_{i'}r_{i'}(x_{i'}-\underline{x}_{i'})\left(1-\frac{\vec{\gamma}_{i'}\vec{x}}{\kappa_{i'}}\right) \tag{99}$$

The corresponding optimality conditions are:

$$\frac{\partial\mathscr{H}_c}{\partial n_k} = 0 \tag{100}$$

$$\dot{x}_i = r_i(x_i-\underline{x}_i)\left(1-\frac{\vec{\gamma}_i\vec{x}_t}{\kappa_i}\right) - n_k h_{ik}(\vec{x}) \tag{101}$$

$$\dot{\mu}_i = {}^t\rho\mu_i - \frac{\partial\mathscr{H}_c}{\partial x_i} \tag{102}$$

$$x_{i\underline{t}} = x_{i1} \tag{103}$$

$$\mu_{i\bar{t}} = 0 \tag{104}$$

For (102) the partial derivative of the Hamiltonian with respect to the stock levels is: 1

$$\frac{\partial\mathscr{H}_c}{x_i} = \alpha\left(\sum_{i=1}^{\bar{i}}(h_{ik}(\vec{x})n_k)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i'=1}^{\bar{i}}\left[(h_{i'k}(\vec{x})n_k)^{\frac{\sigma-1}{\sigma}-1}n_k\frac{\partial h_{i'k}(\vec{x}}{\partial x_i}\right] + \sum_{i''=1}^{\bar{i}}\left[\mu_{i''}\frac{\partial g_{i''}(\vec{x})}{\partial x_i} - n_k\mu_{i''}\frac{\partial h_{i''k}\vec{x}}{\partial x_i}\right]$$

using $h_{ik}(\vec{x}) = \chi_i\nu_{ik}e_k^*$ and $\frac{\partial h_{i'k}(\vec{x})}{\partial x_i} = \frac{\partial\chi_{i'}}{\partial x_i}\nu_{ik}e_k^*$ and rearanging a bit

$$\Leftrightarrow \quad \frac{\partial\mathscr{H}_c}{x_i} = \alpha\left(\sum_{i=1}^{\bar{i}}(\chi_i\nu_{ik}e_k^*n_k)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i'=1}^{\bar{i}}\left[(\chi_{i'}\nu_{i'k}e_k^*n_k)^{\frac{\sigma-1}{\sigma}-1}n_k\frac{\partial\chi_{i'}}{\partial x_i}\nu_{i'k}e_k^*\right]$$
$$+\sum_{i''=1}^{\bar{i}}\left[\mu_{i''}\frac{\partial g_{''i}(\vec{x})}{\partial x_i} - n_k\mu_{i''}\frac{\partial\chi_{i''}}{\partial x_i}\nu_{i''k}e_k^*\right]$$

expand by χ_i

$$\Leftrightarrow \quad \frac{\partial \mathscr{H}_c}{x_i} = \alpha \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{ik} e_k^* n_k)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i'=1}^{\bar{i}} \left[(\chi_{i'} \nu_{i'k} e_k^* n_k)^{\frac{\sigma-1}{\sigma}-1} n_k \frac{\partial \chi_{i'}}{\partial x_i} \nu_{i'k} e_k^* \chi_{i'} \chi_{i'}^{-1} \right]$$
$$+ \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \frac{\partial g''_i(\vec{x})}{\partial x_i} - n_k \mu_{i''} \frac{\partial \chi_{i''}}{\partial x_i} \nu_{i''k} e_k^* \right]$$

$$\Leftrightarrow \quad \frac{\partial \mathscr{H}_c}{x_i} = \alpha \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{ik} e_k^* n_k)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i'=1}^{\bar{i}} \left[(\chi_{i'} \nu_{i'k} e_k^* n_k)^{\frac{\sigma-1}{\sigma}} \frac{\partial \chi_{i'}}{\partial x_i} \chi_{i'}^{-1} \right]$$
$$+ \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \frac{\partial g''_i(\vec{x})}{\partial x_i} - n_k \mu_{i''} \frac{\partial \chi_{i''}}{\partial x_i} \nu_{i''k} e_k^* \right]$$

$$\Leftrightarrow \quad \frac{\partial \mathscr{H}_c}{x_i} = \alpha \left(\sum_{i=1}^{\bar{i}} (h_{ik}(\vec{x}) n_k)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i'=1}^{\bar{i}} \left[(h_{i'k}(\vec{x}) n_k)^{\frac{\sigma-1}{\sigma}} \frac{\partial \chi_{i'}}{\partial x_i} \chi_{i'}^{-1} \right]$$
$$+ \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \frac{\partial g''_i(\vec{x})}{\partial x_i} - n_k \mu_{i''} \frac{\partial \chi_{i''}}{\partial x_i} \nu_{i''k} e_k^* \right]$$

Now plug into (102)

$$\Leftrightarrow \quad \dot{\mu}_i = {}^t\rho \mu_i - \alpha \left(\sum_{i'''=1}^{\bar{i}} (h_{i'''k}(\vec{x}) n_k)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i'=1}^{\bar{i}} \left[(h_{i'k}(\vec{x}) n_k)^{\frac{\sigma-1}{\sigma}} \frac{\partial \chi_{i'}}{\partial x_i} \chi_{i'}^{-1} \right]$$
$$- \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \frac{\partial g''_i(\vec{x})}{\partial x_i} - n_k \mu_{i''} \frac{\partial \chi_{i''}}{\partial x_i} \nu_{i''k} e_k^* \right]$$

Using the steady state assumption

$$\Leftrightarrow \quad 0 = {}^t\rho \mu_i - \underbrace{\alpha \left(\sum_{i'''=1}^{\bar{i}} (g_{i'''}(\vec{x}))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1} \sum_{i'=1}^{\bar{i}} \left[(g_{i'}(\vec{x}))^{\frac{\sigma-1}{\sigma}} \frac{\partial \chi_{i'}}{\partial x_i} \chi_{i'}^{-1} \right]}_{{}^{\mu}b_i(\vec{x})}$$
$$+ \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \left(g_{i''}(\vec{x}) \frac{\partial \chi_{i''}}{\partial x_i} \chi_{i''}^{-1} - \frac{\partial g''_i(\vec{x})}{\partial x_i} \right) \right]$$

$$\Leftrightarrow \quad {}^{\mu}b_i(\vec{x}) = {}^t\rho \mu_i + \sum_{i''=1}^{\bar{i}} \left[\mu_{i''} \left(g_{i''}(\vec{x}) \frac{\partial \chi_{i''}}{\partial x_i} \chi_{i''}^{-1} - \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right) \right]$$

$$\Leftrightarrow \quad {}^{\mu}b_i(\vec{x}) = \mu_i \left({}^t\rho + g_i(\vec{x}) \frac{\partial \chi_i}{\partial x_i} \chi_i^{-1} - \frac{\partial g_i(\vec{x})}{\partial x_i} \right) + \sum_{\substack{i''=1 \\ i'' \neq i}}^{\bar{i}} \left[\mu_{i''} \left(g_{i''}(\vec{x}) \frac{\partial \chi_{i''}}{\partial x_i} \chi_{i''}^{-1} - \frac{\partial g_{i''}(\vec{x})}{\partial x_i} \right) \right] \tag{105}$$

There are $\bar{i}$ of these equations to solve as a linear system of equations they can be expressed simultaneously in the form

$$ {}^{\mu}A\vec{\mu} = {}^{\mu}\vec{b} \tag{106}$$

with the solution

$$\vec{\mu} = {}^{\mu}A^{-1}\, {}^{\mu}\vec{b} \tag{107}$$

with ${}^{\mu}A(\vec{x}) \in \mathbb{R}^{\bar{i} \times \bar{i}}$, ${}^{\mu}\vec{b}(\vec{x}) \in \mathbb{R}^{\bar{i}}$ and $\vec{\mu} \in \mathbb{R}^{\bar{i}}$.

Let the elements of ${}^{\mu}A$ be given by ${}^{\mu}a_{ii'}$ and those of ${}^{\mu}\vec{b}$ by ${}^{\mu}b_i$.

$$
\begin{aligned}
{}^{\mu}a_{ii} &= {}^{t}\rho + g_i(\vec{x})\frac{\partial \chi_i}{\partial x_i}\chi_i^{-1} - \frac{\partial g_i(\vec{x})}{\partial x_i} \\
{}^{\mu}a_{ii'} &= g_{i'}(\vec{x})\frac{\partial \chi_{i'}}{\partial x_i}\chi_{i'}^{-1} - \frac{\partial g_{i'}(\vec{x})}{\partial x_i} \qquad i \neq i' \\
{}^{\mu}b_i &= \alpha\left(\sum_{i''=1}^{\bar{i}}\left(g_{i''}(\vec{x})\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i'''=1}^{\bar{i}}\left[\left(g_{i'''}(\vec{x})\right)^{\frac{\sigma-1}{\sigma}}\frac{\partial \chi_{i'''}}{\partial x_i}\chi_{i'''}^{-1}\right]
\end{aligned}
$$

This can be simplified by using that $\frac{\partial \chi_i}{x_{i'}} = 0$ for $i \neq i'$.

$$
\begin{aligned}
{}^{\mu}a_{ii} &= {}^{t}\rho + g_i(\vec{x})\frac{\partial \chi_i}{\partial x_i}\chi_i^{-1} - \frac{\partial g_i(\vec{x})}{\partial x_i} \\
{}^{\mu}a_{ii'} &= -\frac{\partial g_{i'}(\vec{x})}{\partial x_i} \qquad i \neq i' \\
{}^{\mu}b_i &= \alpha\left(\sum_{i''=1}^{\bar{i}}\left(g_{i''}(\vec{x})\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\left(g_i(\vec{x})\right)^{\frac{\sigma-1}{\sigma}}\frac{\partial \chi_i}{\partial x_i}\chi_i^{-1}
\end{aligned}
$$

Rearranging (100) and using μ as derived above, the demand function in this case is the defined as follows:

$$
\begin{aligned}
\frac{\partial \mathscr{H}_c}{\partial n_k} &= -c_k + \alpha\left(\sum_{i=1}^{\bar{i}}\left(h_{ik}(\vec{x})n_k\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}\sum_{i''=1}^{\bar{i}}h_{i''k}(\vec{x})\left(h_{i''k}(\vec{x})n_k\right)^{\frac{\sigma-1}{\sigma}-1} \\
&\quad - \sum_{i'=1}^{\bar{i}}\mu_{i'}h_{i'k}(\vec{x}) \\
\Leftrightarrow \qquad 0 &= -c_k + \alpha n_k^{-\frac{1}{\eta}}\left(\sum_{i=1}^{\bar{i}}\left(h_{ik}(\vec{x})n_k\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} - \sum_{i''=1}^{\bar{i}}\mu_{i'}h_{i'k}(\vec{x}) \\
\Leftrightarrow \qquad c_k + \sum_{i''=1}^{\bar{i}}\mu_{i'}h_{i'k}(\vec{x}) &= \alpha n_k^{-\frac{1}{\eta}}\left(\sum_{i=1}^{\bar{i}}\left(h_{ik}(\vec{x})n_k\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}} \\
\Leftrightarrow \qquad n_k &= \alpha^{\eta}\left(c_k + \sum_{i''=1}^{\bar{i}}\mu_{i'}h_{i'k}(\vec{x})\right)^{-\eta}\left(\sum_{i=1}^{\bar{i}}\left(h_{ik}(\vec{x})n_k\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{(\eta-1)\sigma}{\sigma-1}} \qquad (108)
\end{aligned}
$$

C.3 Switching Between Cases

The condition for switching between the two cases above can be found using the optimality conditions of the second case and the Kuhn-Tucker conditions for non-negativity of Firms.

$$
\max\int_{t_1}^{\bar{t}}\omega - \sum_{k=1}^{\bar{k}}n_k c_k + \alpha\frac{\eta}{\eta-1}\left(\sum_{i=1}^{\bar{i}}\left(\sum_{k'=1}^{\bar{k}}n_{k'}h_{ik'}(\vec{x})\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}}\mathrm{e}^{-{}^{t}\rho t}\mathrm{d}t \qquad (109)
$$

subject to

$$
\dot{x}_i = g(\vec{x}) - \sum_{k=1}^{\bar{k}}n_k h_{ik}(\vec{x}) \qquad (110)
$$

$$
-n_k \leq 0 \qquad (111)
$$

The Hamiltonian for this problem is given by:

$$\mathscr{H}_c = \omega - \sum_{k=1}^{\bar{k}} n_k c_k + \alpha \frac{\eta}{\eta - 1} \left(\sum_{i=1}^{\bar{i}} \left(\sum_{k'=1}^{\bar{k}} n_{k'} h_{ik'}(x_i) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta}} + \sum_{k'''=1}^{\bar{k}} \lambda_{k'''} n_{k'''} + \sum_{\imath'=1}^{\bar{i}} \mu_{i'} \left(g_{i'}(\vec{x}) - \sum_{k''=1}^{\bar{k}} n_{k''} h_{i'k''}(x_{i'}) \right) \tag{112}$$

The corresponding first order conditions are then:

$$\frac{\partial \mathscr{H}_c}{\partial n_k} = 0 \tag{113}$$

$$\dot{x}_i = g(\vec{x}) - \sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \tag{114}$$

$$\dot{\mu}_i = {}^t\rho \mu_i - \sum_{k=1}^{\bar{k}} n_k h_{ik}(x_i) \tag{115}$$

The transversality conditions are:

$$x_{it} = x_{i0} \tag{116}$$

$$\mu_{i\bar{t}} = 0 \tag{117}$$

The Kuhn-Tucker conditions are:

$$\lambda_k \geq 0 \tag{118}$$

$$-n_k \leq 0 \tag{119}$$

$$-\lambda_k n_k = 0 \tag{120}$$

The condition for switching between cases is found by using condition (120) in conjunction with (100) under assumption that $\bar{i} = \bar{k} = 2$ for case 2 that is one firm being active, as the sole active firm, and the other is not.

Let $n_1 = 0$, $n_2 > 0$, and hence $\lambda_1 \geq 0$ and $\lambda_2 = 0$.

Starting from (100) for Tool 1:

$$\begin{aligned} 0 = -c_1 + \alpha \left(\sum_{i=1}^{\bar{i}} \left(\sum_{k'=1}^{\bar{k}} n_{k'} h_{ik'}(x_i) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta} - 1} \sum_{i''=1}^{\bar{i}} \left[h_{i''1}(x_{i''}) \left(\sum_{k''=1}^{\bar{k}} n_{k''} h_{i''k''}(x_{i''}) \right)^{\frac{\sigma-1}{\sigma} - 1} \right] \\ + \lambda_1 - \sum_{i'=1}^{\bar{i}} [\mu_{i'} h_{i'1}(x_{i'})] \end{aligned}$$

Substituting the n_k as specified above.

$$\Rightarrow \quad 0 = -c_1 + \alpha \left(\sum_{i=1}^{\bar{i}} (n_2 h_{i2}(x_i))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta} - 1} \sum_{i''=1}^{\bar{i}} \left[h_{i''1}(x_{i''}) \left(n_2 h_{i''2}(x_{i''}) \right)^{\frac{\sigma-1}{\sigma} - 1} \right] + \lambda_1 - \sum_{i'=1}^{\bar{i}} [\mu_{i'} h_{i'1}(x_{i'})]$$

Using $\lambda_1 \geq 0$.

$$\Leftrightarrow \quad 0 \geq n_2^{-\frac{1}{\eta}} \alpha \left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \frac{\eta-1}{\eta} - 1} \sum_{i''=1}^{\bar{i}} \left[h_{i''1}(x_{i''}) h_{i''2}(x_{i''})^{\frac{\sigma-1}{\sigma} - 1} \right] - c_1 - \sum_{i'=1}^{\bar{i}} [\mu_{i'} h_{i'1}(x_{i'})]$$

Substitute n_2 by (108). This is possible due to the assumption of Tool 2 being the only tool in use.

$$\Rightarrow \quad 0 \geq \overbrace{\left(c_2 + \sum_{i'''=1}^{\bar{i}} \mu_{i'''} h_{i'''2}(x_{i'''})\right) \alpha^{-1} \left(\sum_{i^{iv}=1}^{\bar{i}} h_{i^{iv}2}(x_{i^{iv}})^{\frac{\sigma-1}{\sigma}}\right)^{-\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}}}^{n_2^{-\frac{1}{\eta}}} \alpha \left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\eta-1}{\eta}-1}$$
$$\sum_{i''=1}^{\bar{i}} \left[h_{i''1}(x_{i''}) h_{i''2}(x_{i''})^{\frac{\sigma-1}{\sigma}-1}\right] - c_1 - \sum_{i'=1}^{\bar{i}} [\mu_{i'} h_{i'1}(x_{i'})]$$
$$\Leftrightarrow \quad 0 \geq \left(c_2 + \sum_{i'''=1}^{\bar{i}} \mu_{i'''} h_{i'''2}(x_{i'''})\right) \left(\sum_{i=1}^{\bar{i}} h_{i2}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[h_{i''1}(x_{i''}) h_{i''2}(x_{i''})^{\frac{\sigma-1}{\sigma}-1}\right] - c_1 - \sum_{i'=1}^{\bar{i}} [\mu_{i'} h_{i'1}(x_{i'})]$$

Substitute minimum harvesting costs (20) and equilibrium harvests (6) with (9) for c_k and $h_{ik}(x_i)$.

$$\Rightarrow \quad 0 \geq \left(\phi_2 + \omega e_2^* + \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2} e_2^{*\varepsilon}]\right) \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2} e_2^{*\varepsilon})^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} e_1^{*\varepsilon} (\chi_{i''} \nu_{i''2} e_2^{*\varepsilon})^{\frac{\sigma-1}{\sigma}-1}\right]$$
$$- \phi_1 - \omega e_1^* - \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1} e_1^{*\varepsilon}]$$
$$\Leftrightarrow \quad 0 \geq e_1^{*\varepsilon} e_2^{*-\varepsilon} \left(\phi_2 + \omega e_2^* + e_2^{*\varepsilon} \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2}]\right) \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} (\chi_{i''} \nu_{i''2})^{\frac{\sigma-1}{\sigma}-1}\right]$$
$$- \phi_1 - \omega e_1^* - e_1^{*\varepsilon} \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1}]$$
$$\Leftrightarrow \quad 0 \geq \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} (\chi_{i''} \nu_{i''2})^{\frac{\sigma-1}{\sigma}-1}\right] - \frac{e_1^{*-\varepsilon} \left(\phi_1 + \omega e_1^* + e_1^{*\varepsilon} \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1}]\right)}{e_2^{*-\varepsilon} \left(\phi_2 + \omega e_2^* + e_2^{*\varepsilon} \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2}]\right)}$$

Substitute (9) for e_k^* and rearrange.

$$\Rightarrow \quad 0 \geq \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} (\chi_{i''} \nu_{i''2})^{\frac{\sigma-1}{\sigma}-1}\right] - \frac{\frac{\phi_1^{1-\varepsilon}}{\omega^{-\varepsilon}} \frac{\varepsilon^{-\varepsilon}}{(1-\varepsilon)^{1-\varepsilon}} \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1}]}{\frac{\phi_2^{1-\varepsilon}}{\omega^{-\varepsilon}} \frac{\varepsilon^{-\varepsilon}}{(1-\varepsilon)^{1-\varepsilon}} \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2}]}$$
$$\Leftrightarrow \quad 0 \geq \left(\sum_{i=1}^{\bar{i}} (\chi_i \nu_{i2})^{\frac{\sigma-1}{\sigma}}\right)^{-1} \sum_{i''=1}^{\bar{i}} \left[\chi_{i''} \nu_{i''1} (\chi_{i''} \nu_{i''2})^{\frac{\sigma-1}{\sigma}-1}\right] - \frac{\phi_1^{1-\varepsilon} \sum_{i'=1}^{\bar{i}} [\mu_{i'} \chi_{i'} \nu_{i'1}]}{\phi_2^{1-\varepsilon} \sum_{i'''=1}^{\bar{i}} [\mu_{i'''} \chi_{i'''} \nu_{i'''2}]} \tag{121}$$

C.4 Conditions For No Effect of Bycatch under Optimal Harvesting

Without loss of generality the derivation is performed for the shadow price of species 1 to changes in the catch composition of harvesting gear type 2. Shadow prices of a single species, when ecosystem interaction is not present is given by (28).

$$\mu_i = \left({}^t\rho - \frac{\partial g_i(\vec{x})}{\partial x_i}\right)^{-1} \left(-\frac{\partial p_i(x_i)}{\partial x_i} g_i(\vec{x})\right) \tag{28}$$

The derivation of the shadow price to the individual harvesting efficiency $\nu_i' k$ is then given by the following equation.

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}\nu_{i'k}} = \left({}^t\rho - \frac{\partial g_i(\vec{x})}{\partial x_i}\right)^{-1} \left(-\frac{\partial \frac{\partial p_i(x_i)}{\partial x_i}}{\partial \nu_{i'k}} g_i(\vec{x})\right)$$

This is plugged in to the equality between different derivatives of shadow prices to changing harvesting efficiencies.

$$\frac{\mathrm{d}\mu_1}{\mathrm{d}\nu_{12}} = \frac{\mathrm{d}\mu_1}{\mathrm{d}\nu_{22}}$$
$$\left({}^t\rho - \frac{\partial g_1(\vec{x})}{\partial x_1}\right)^{-1}\left(-\frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{12}} g_1(\vec{x})\right) = \left({}^t\rho - \frac{\partial g_1(\vec{x})}{\partial x_1}\right)^{-1}\left(-\frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{22}} g_1(\vec{x})\right)$$
$$\frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{12}} = \frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{22}}$$

The price derivative to stock change is given by the following equation.

$$\frac{\partial p_1(x_1)}{\partial x_1} = -\chi_1^{-2}(\nu_{11}\nu_{22} - \nu_{12}\nu_{21})^{-1}(\nu_{22}\,{}^pb_1 - \nu_{21}\,{}^pb_2) \tag{122}$$

Plugging this into the expression above then yields the same condition for no effect of bycatch as in the open-access case.

$$\frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{12}} = \frac{\partial \frac{\partial p_1(x_1)}{\partial x_1}}{\partial \nu_{22}} \tag{123}$$
$$\frac{\nu_{21}\left({}^pb_1\nu_{22} - {}^pb_2\nu_{21}\right)}{(\nu_{12}\nu_{21} - \nu_{11}\nu_{21})^2\,(-\chi_1^2)} = \frac{\nu_{21}\left({}^pb_2\nu_{11} - {}^pb_1\nu_{12}\right)}{(\nu_{12}\nu_{21} - \nu_{11}\nu_{22})^2\,(-\chi_1^2)}$$
$${}^pb_1\nu_{22} - {}^pb_2\nu_{21} = {}^pb_2\nu_{11} - {}^pb_1\nu_{12}$$
$${}^pb_1(\nu_{22} + \nu_{12}) = {}^pb_2(\nu_{11} + \nu_{21})$$
$$\frac{{}^pb_1}{{}^pb_2} = \frac{\nu_{11} + \nu_{21}}{\nu_{12} + \nu_{22}} \tag{124}$$

substitute ${}^pb$ with (59)

$$\frac{\phi_1\left(1 + \frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon}}{\phi_k\left(1 + \frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{\phi_k}{\omega}\frac{\varepsilon}{1-\varepsilon}\right)^{-\varepsilon}} = \frac{\nu_{11} + \nu_{21}}{\nu_{12} + \nu_{22}}$$
$$\frac{\phi_1^{1-\varepsilon}}{\phi_2^{1-\varepsilon}} = \frac{\nu_{11} + \nu_{21}}{\nu_{12} + \nu_{22}}$$
$$\frac{\phi_1}{\phi_2} = \left(\frac{\nu_{11} + \nu_{21}}{\nu_{12} + \nu_{22}}\right)^{\frac{1}{1-\varepsilon}} \tag{125}$$

D Potential Mortality under TAC with Discarding

The derivation of potential mortality under TAC goes as follows. For the derivation it is assumed that q_1 is bound by the TAC and q_2 is harvested unrestricted through discarding. Both species are harvested using the same gear k and effort level e_k

$$q_1 = \nu_{1k}\chi_1 n_k e_k^{\varepsilon} \tag{126}$$
$$q_2 = \nu_{2k}\chi_2 n_k e_k^{\varepsilon} \tag{127}$$

rearanging to isolate the identical fleet size and harvesting effort determingn quantities and setting both to be equal

$$q_1\nu_{1k}^{-1}\chi_1^{-1} = q_2\nu_{2k}^{-1}\chi_2^{-1} \tag{128}$$
$$q_2 = q_1\frac{\nu_{2k}\chi_2}{\nu_{1k}\chi_1} \tag{129}$$

References

Abbott, Joshua K and James E Wilen (2009), "Regulation of fisheries bycatch with common-pool output quotas." *Journal of Environmental Economics and Management*, 57, 195–204.

Asche, Frank, Frode Steen, and Kjell G Salvanes (1997), "Market delineation and demand structure." *American Journal of Agricultural Economics*, 79, 139–150.

Barten, Anton P and Leon J Bettendorf (1989), "Price formation of fish: An application of an inverse demand system." *European Economic Review*, 33, 1509–1525.

Baumgärtner, Stefan, Sandra Derissen, Martin F Quaas, and Sebastian Strunz (2011), "Consumer preferences determine resilience of ecological-economic systems." *Ecology & Society*, 16.

Beddington, John R, David J Agnew, and Colin W Clark (2007), "Current problems in the management of marine fisheries." *science*, 316, 1713–1716.

Blanz, Benjamin (2018), "Modelling interactions of fish, fishers and consumers: should bycatch be taken into account?" *Hydrobiologia*, 1–16.

Bose, Shekar and Alistair McIlgorm (1996), "Substitutability among species in the japanese tuna market: a cointegration analysis." *Marine Resource Economics*, 11, 143–155.

Chiang, Fu-Sung, Jonq-Ying Lee, Mark G Brown, et al. (2001), "The impact of inventory on tuna price: An application of scaling in the rotterdam inverse demand system." *Journal of Agricultural and Applied Economics*, 33, 403–412.

Da Rocha, José-María, Santiago Cervino, and Sebastian Villasante (2012), "The common fisheries policy: an enforcement problem." *Marine Policy*, 36, 1309–1314.

Daan, Niels (1997), "Tac management in north sea flatfish fisheries." *Journal of Sea Research*, 37, 321–341.

Davies, RWD, SJ Cripps, A Nickson, and G Porter (2009), "Defining and estimating global marine fisheries bycatch." *Marine Policy*, 33, 661–672.

Dixit, Avinash K and Joseph E Stiglitz (1977), "Monopolistic competition and optimum product diversity." *The American Economic Review*, 67, 297–308.

Fulton, Elizabeth A, Anthony DM Smith, David C Smith, and Ingrid E van Putten (2011), "Human behaviour: the key source of uncertainty in fisheries management." *Fish and Fisheries*, 12, 2–17.

Gulland, JA and Serge Garcia (1984), "Observed patterns in multispecies fisheries." In *Exploitation of marine communities*, 155–190, Springer.

Hollowed, Anne B, Nicholas Bax, Richard Beamish, Jeremy Collie, Michael Fogarty, Patricia Livingston, John Pope, and Jake C Rice (2000), "Are multispecies models an improvement on single-species models for measuring fishing impacts on marine ecosystems?" *ICES Journal of Marine Science*, 57, 707–719.

Kuhn, Harold W (2014), "Nonlinear programming: a historical view." In *Traces and Emergence of Nonlinear Programming*, 393–414, Birkhäuser, Basel.

Larkin, Peter A (1977), "An epitaph for the concept of maximum sustained yield." *Transactions of the American fisheries society*, 106, 1–11.

Link, Jason S (2002a), "Ecological considerations in fisheries management: when does it matter?" *Fisheries*, 27, 10–17.

Link, Jason S (2002b), "What does ecosystem-based fisheries management mean." *Fisheries*, 27, 18–21.

May, Robert M, John R Beddington, Colin W Clark, Sidney J Holt, and Richard M Laws (1979), "Management of multispecies fisheries." *Science*, 205, 267–277.

Nieminen, Emmi, Marko Lindroos, and Outi Heikinheimo (2012), "Optimal bioeconomic multispecies fisheries management: a baltic sea case study." *Marine Resource Economics*, 27, 115–136.

Pearl, Raymond and Lowell J Reed (1977), "On the rate of growth of the population of the united states since 1790 and its mathematical representation." In *Mathematical Demography*, 341–347, Springer.

Pelletier, Dominique, Stéphanie Mahevas, Hilaire Drouineau, Youen Vermard, Olivier Thebaud, Olivier Guyader, and Benjamin Poussin (2009), "Evaluation of the bioeconomic sustainability of multi-species multi-fleet fisheries under a wide range of policy options using isis-fish." *Ecological Modelling*, 220, 1013–1033.

Pigou, AC (1920), "The economics of welfare, lst ed."

Pikitch, Ellen K, C Santora, E A Babcock, A Bakun, R Bonfil, D O Conover, P Dayton, P Doukakis, D Fluharty, B Heneman, et al. (2004), "Ecosystem-based fishery management." *Science*, 305, 346–347.

Plagányi, Éva E, André E Punt, Richard Hillary, Elisabetta B Morello, Olivier Thébaud, Trevor Hutton, Richard D Pillans, James T Thorson, Elizabeth A Fulton, Anthony DM Smith, et al. (2014), "Multispecies fisheries management and conservation: tactical applications using models of intermediate complexity." *Fish and Fisheries*, 15, 1–22.

Quaas, Martin F and Till Requate (2013), "Sushi or fish fingers? seafood diversity, collapsing fish stocks, and multispecies fishery management." *The Scandinavian Journal of Economics*, 115, 381–422.

Singh, Rajesh and Quinn Weninger (2009), "Bioeconomies of scope and the discard problem in multiple-species fisheries." *Journal of Environmental Economics and Management*, 58, 72–92.

Skonhoft, Anders, Niels Vestergaard, and Martin F. Quaas (2012), "Optimal harvest in an age structured model with different fishing selectivity." *Environmental and Resource Economics*, 51, 525–544, URL `http://dx.doi.org/10.1007/s10640-011-9510-x`.

Squires, Dale, Harry Campbell, Stephen Cunningham, Christopher Dewees, R Quentin Grafton, Samuel F Herrick, James Kirkley, Sean Pascoe, Kjell Salvanes, Bruce Shallard, et al. (1998), "Individual transferable quotas in multispecies fisheries." *Marine Policy*, 22, 135–159.

Ströbele, WJ and H Wacker (1991), "The concept of sustainable yield in multi-species fisheries." *Ecological Modelling*, 53, 61–74.

Sutinen, Jon G and Peder Andersen (1985), "The economics of fisheries law enforcement." *Land economics*, 61, 387–397.

Yodzis, Peter (1994), "Predator-prey theory and management of multispecies fisheries." *Ecological applications*, 4, 51–58.